

Introduction aux Théories de la Gravitation

Cours: Physique Master 1 (ENS)



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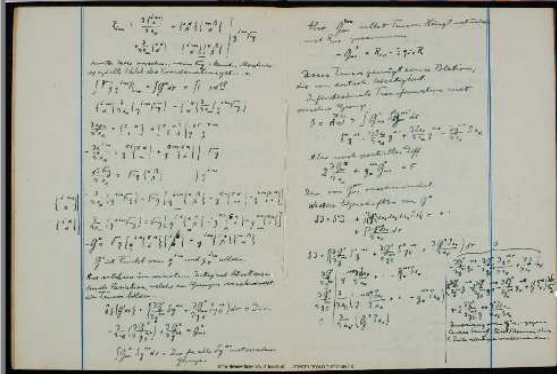
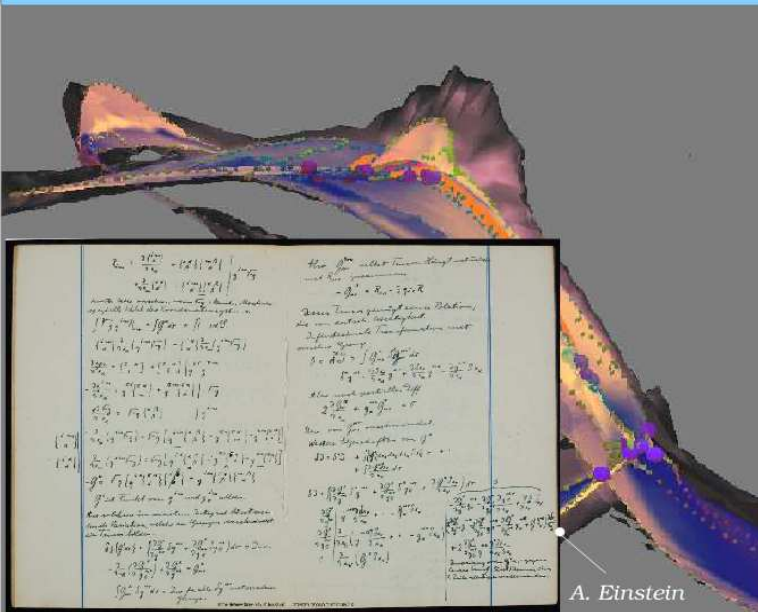
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**Introduction aux
Théories de la Gravitation**


Éléments de
Physique Théorique et de Géométrie




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A. Einstein



Lyon 1



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LITTÉRATURE

Relativité Générale:

P. A.M. Dirac: *General Theory of Relativity*, Princeton Landmarks in Physics, Princeton Univ. Press, 1996.

R. M. Wald: *General Relativity*, University of Chicago Press, 1984.

L. P. Hughston und K. P. Tod: *An Introduction to General Relativity*, London Mathematical Society Student Texts, Cambridge Univ. Press, 1991.

W. Rindler: *Essential Relativity. Special, General, and Cosmological*, Springer Berlin, 1977.

C. W. Misner, K. S. Thorne und J. A. Wheeler: *Gravitation*, W. H. Freeman & Co Ltd, 1973.

Formulation de Cartan / Formes différentielles:

B. Schutz: *Geometrical Methods of Mathematical Physics*, Cambridge Univ. Press, 1980.

C. Rovelli: *Quantum Gravity* (classical part of the book), Cambridge Univ. Press, 2004.

N. Straumann: *General Relativity with Applications to Astrophysics*, Springer, Berlin, 2004.

S. Chandrasekhar: *The Mathematical Theory of Black Holes*, Clarendon Press, 1998.

Développement historique:

A. Pais: *Subtle is the Lord ... The Science and the Life of Albert Einstein*, Oxford Univ. Press, 1982.

Archives d'Internet:

NASA-ADS: <http://adswww.harvard.edu/>

HEP-SPIRES: <http://www.slac.stanford.edu/spires/>

Preprint arXiv: <http://de.arxiv.org/>

Les notes originales d'Einstein: <http://www.alberteinstein.info/manuscripts/>