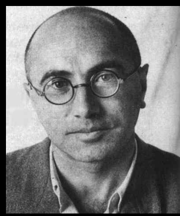
Newton

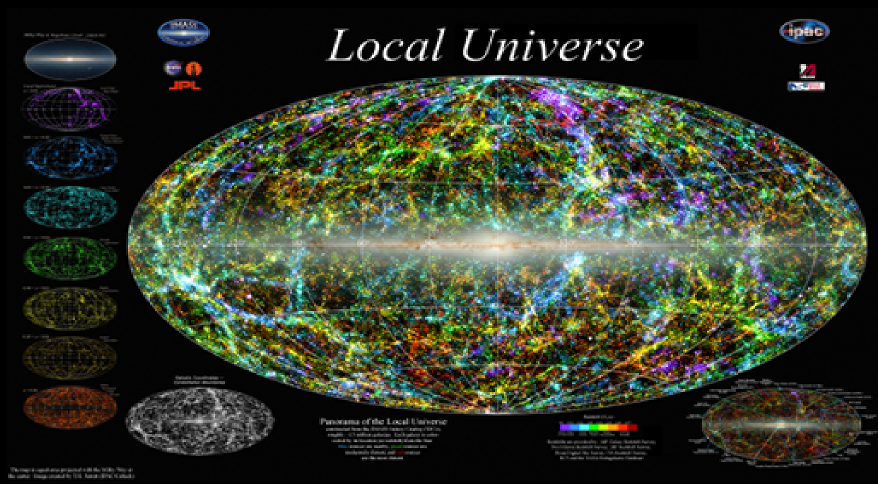
T. Buchert:

COSMOLOGIE & SYSTÈMES GRAVITATIONNELS



Zel'dovich

Cours : Science de la Matière — Master 2 (ENS)



Chandrasekhar

Ellis

Contents

The paragraphs marked with "*TD*" are exercises that are distributed during the lectures. Their solutions will be given within the lecture notes, distributed at the end of each chapter.

Selected Books

C/: Cosmology **A/**: Astrophysics **S/**: Statistical Mechanics **P/**: Plasmaphysics

C/ F. Adams, T. Buchert, L. Mersini–Houghston: *Cosmic Update: dark puzzles, the arrow of time, future history*, Springer New York: Multiversal Journeys II, ed. by F. Nekogaar, 2011.

S/ R. Balescu: *Equilibrium and Nonequilibrium Statistical Mechanics*, Wiley, New York, 1975.

A/ J. Binney, S. Tremaine: *Galactic dynamics*, Princeton University Press, 1994.

C/ G. Börner: *The Early Universe – Facts and Fiction*, 4^{th} edition, Springer Berlin, 2003.

S/ S. Chapman, T.G. Cowling: *The mathematical theory of non–uniform gases*, Cambridge Univ. Press, 1939.

A/ S. Chandrasekhar: *An introduction to the study of stellar structure*, Dover, 1967.

A/ S. Chandrasekhar: *Ellipsoidal figures of equilibrium*, Dover, 1969.

C/ E.R. Harrison: *Cosmology: the science of the Universe*, Cambridge University Press, 2000.

P/ E.H. Holt, R.E. Haskell: *Foundations of plasma dynamics*, Macmillan, N.Y., 1968.

S/ J. Jeans: *The dynamical theory of gases*, 4th ed., Cambridge University Press, 1954.

C/ E.W. Kolb, M. Turner: *The Early Universe*, Westview Press, 1994.

S/ R. L. Liboff: *Introduction to the theory of kinetic equations*, Krieger Pub. Com., Huntington, NY, 1979.

C/ V.F. Mukhanov: *Physical foundations of cosmology*, Cambridge University Press, 2005.

C/ M.K. Munitz: *Theories of the Universe*, Simon and Schuster, 1965.

C/ J.V. Narlikar: *An introduction to cosmology*, 3rd ed., Cambridge University Press, 2002.

A/ K.F. Ogorodnikov: *Dynamics of stellar systems*, Pergamon Press, 1965.

C/ T. Padmanabhan: *Structure formation in the Universe*, Cambridge University Press, 1993.

C/ J.A. Peacock: *Cosmological physics*, Cambridge University Press, 1999.

C/ P.J.E. Peebles: *Large Scale Structure of the Universe*, Princeton University Press, 1980.

S/ P. Résibois, M. de Leener: *Classical kinetic theory of fluids*, Wiley, 1977.

P/ I.P. Shkarofsky, T.W. Johnston, M.P. Bachynski: *The particle kinetics of plasmas*, Addison Wesley, 1966.

A/ W.C. Saslaw: *Gravitational physics of stellar and galactic systems*, Cambridge University Press, 1985.

C/ S. Weinberg: *Cosmology*, Oxford University Press, 2008.

P/ T.Y. Wu: *Kinetic equations of gases and plasmas*, Addison Wesley, 1966.

C/ Ya.B. Zel'dovich, I.D. Novikov: *Relativistic Astrophysics: The structure and evolution of the Universe*, University of Chicago Press, 1983.

OVERVIEW

We shall restrict these lectures to the Newtonian theory of gravitation, since only there the understanding of the complexity of self–gravitating systems has been developed to some depth, which is of interest to us here. Also, the standard model of cosmology, perturbation theories and theories for structure formation are essentially based on the Newtonian theory. This is especially true for the Late Universe to which we focus our attention here. The idea of this course is not to give details of standard concepts in cosmology, that can be easily found in the literature, but to provide the physical foundations that also furnish the basis to go beyond standard concepts. We shall study the complexity of phenomena on all spatial scales where gravity is known to rule physical systems, from solar system scales to stellar systems, galactic, supergalactic and cosmological systems. We are going to illustrate this scale–dependence in terms of observational results.

After introducing our basic framework, we shall commence with systems on cosmological scales and subsequently refine our description to access smaller spatial scales. Most concepts developed here are also useful for the building of more general theories of gravitation (like Einstein's theory), and they are also relevant for investigations in other fields of theoretical physics. We shall point out and explain, how these concepts provide tools at the interface with other theories, notably Maxwell's theory of electrodynamics, plasma physics, and nonlinear dynamical systems.

We shall first consider continuum (or fluid) notions for the description of self–gravitating systems, and later we move on to a more refined understanding of N–particle systems.

Before we start, let us overview (i) the relations between the *physical theories* involved in the description of gravitational systems, (ii) the *different scales* that gravitational systems occupy in space and (iii) criteria that have to be satisfied by a description of *collisionless gravitational systems* to which we shall confine most of these lectures.

Physical context of gravitational systems

These lectures are based on input from various physical theories. The description of gravitational systems emerges from several disciplines

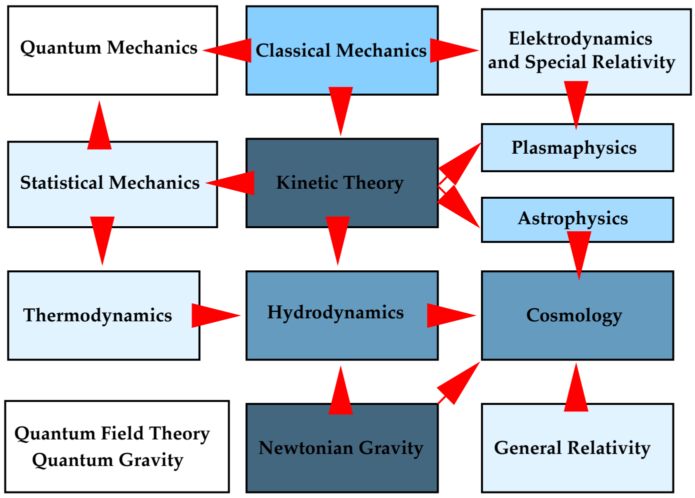


Fig. 0.1. This diagram shows the emphasis of various theories taken in the present course. Darker blocks correspond to more emphasis.

that have been developed for different purposes. But, not only these various disciplines influenced the development of the understanding of gravitational systems: their description keeps a number of formal analogies and can therefore be considered as a basic framework of understanding theoretical concepts in general terms, including the possibility to easily access applications in other fields of research. An overview of the context is given in Fig. 0.1.

Gravitational systems are by their very nature systems that are dominated by the gravitational interaction, which we here describe in Newtonian terms. The detailed developments in Newtonian theory have so far not been fully carried over to the general relativistic context, which is the reason why we are not developing Einstein's gravitation theory in these lectures. We shall, however, put the general relativistic context into perspective. We aim at understanding the complex description of gravitational systems in phase space and, of course, these general concepts will be relevant in the general–relativistic context too, which is a domain of future research — for which you are invited to participate in your eventual research career.

The description of gravitational systems lives at the interface with the kinetic theory of collisionless systems, based on classical mechanics (note that this includes also modern developments on the theory of nonlinear dynamical systems) and being superordered to derived theories like thermodynamics and hydrodynamics. Because Newtonian gravitation bears close formal analogies to classical electrodynamics, the description of N–particle systems in phase space is formally close to the research field of plasma physics. Another interface is furnished with the actual objects to be described with this theory: the astrophysics of stellar, galactic and supergalactic systems, as well as the Universe as a whole. An important application that we shall emphasize in these lectures is the description of gravitational instabilities of cosmological models and nonlinear structure formation. Also here, generic patterns arise that are also relevant to other disciplines of physics.

Spatial scales of gravitational systems

An estimate of the spatial scale on which a given system significantly concentrates matter can be related to the typical fluid density $\varrho := mn$, where m denotes the elementary mass of a "particle", and $n = N/V$ the particle density. On average, such a density defines the spatial scale via the volume V that the system occupies.

Examples:

- The density of galaxies (shining matter) in the visible Universe is about $\bar{\varrho}^{\rm lum}_{\rm gal} \cong 3 \cdot 10^{-31}$g/cm^3.
- the density of all matter may be estimated by $\bar{\varrho} \cong 1,88 \cdot 10^{-29}h_0^2$g/cm^3, where h$_0 = H_0/100$km/Mpc sec is the normalized value of the Hubble expansion today. (This would correspond to about three hydrogene atoms per cubic meter.)
- the density within a galaxy for luminous matter (stars) is about $\bar{\varrho}^{\rm lum}_{\rm stars} \cong 10^{-23}$g/cm^3. (This would correspond to about ten millions of hydrogene atoms per cubic meter.)

The *density contrast* within structures, i.e. a dimensionless excess density over the mean density, defined by $\delta := (\varrho - \bar{\varrho})/\bar{\varrho}$, would volume–average (on the spatial scale of the system) to the following estimated numbers:

- for a region of about 10 Mpc: $\langle\delta\rangle \cong 1$.
- for a typical rich cluster of galaxies: $\langle\delta\rangle \cong 10$.
- for a galaxy: $\langle\delta\rangle \cong 5 \cdot 10^5$.
- for a star: $\langle\delta\rangle \cong 7 \cdot 10^{28}$.
- for the Earth: $\langle\delta\rangle \cong 2,8 \cdot 10^{29}$.
- for the air according to the estimate by Avogadro: $\langle\delta\rangle \cong 2 \cdot 10^{19}$.
- for a human being: $\langle\delta\rangle \cong 2,5 \cdot 10^{28}$.

The *hierarchical structuring* of gravitational systems involves a hierarchy of distance scales that we can measure in terms of their light–distances to us. One usually measures distances in terms of the unit of a parallax second 1 pc (parsec)$= 3,26$ Lj (lightyears) $= 3,086 \cdot 10^{16}$ m.

The following list provides some intuition on the involved distances:

- the time needed for light to travel around the Earth (light distance): d $\cong 1/7$ Lsec.
- the light–distance to the Moon: d $\cong 1,28$ Lsec.
- the light–distance to the Sun: d $\cong 8,3$ Lmin.
- the light–distance to the nearest star: d $\cong 4$ Lj $\approx 1,2$ pc. (Two water melones (the Sun here and another star in Australia), where the Earth would be a pearl 100 m away from the Sun.)
- the diameter of the Milky Way galaxy: d $\cong 10^5$ Lj ≈ 30 Kpc.
- the light–distance to the nearest larger galaxy (Andromeda): d $\cong 2 \cdot 10^6$ Lj ≈ 600 Kpc.
- the light–distance of the Virgo cluster of galaxies to the nearest rich cluster of galaxies Coma: d $\cong 50$ Mpc.
- the light–distance across the largest “void” or supercluster: d $\cong 400$ Mpc.
- the light–distance to cross the visible Universe: d $\cong 6$ Gpc.

The hierarchical structure is also visible in a hierarchy of motions. Typical *peculiar velocities* u, i.e. velocities relative to the *Hubble flow* (and measured in a coordinate system comoving with the Hubble flow), defined later in the lectures, would amount to the typical values:

- the Earth turns around the Sun with a speed of $u \cong 30$ km/sec.
- the Sun turns around the galactic center of the Milky Way with a speed of $u \cong 300$ km/sec.
- the Milky Way itself makes half a turn since the death of the dinosaurs (60 Mio years).

- the Milky Way turns within the Local Group of galaxies (Andromeda etc.) with a speed of $u \cong 100$ km/sec.
- the Local Group of galaxies falls onto the Virgo galactic cluster center of mass with a speed of $u \cong 200$ km/sec.
- the Virgo cluster of galaxies falls onto the Coma rich galactic cluster center of mass with about the same speed where here the motion interfers with the Hubble velocity of the universal expansion.
- Translating the magnitude of the Cosmic Microwave Background dipol fully into a relative motion with respect to the Hubble flow, we obtain an "absolute" motion with speed $u \cong 600$ km/sec towards the Hydra Centaurus cluster agglomeration.

Collisionless kinetics of gravitational systems

A system of N "particles" may be macroscopically described by a scalar density $n = N/V$, where V denotes the occupied volume in space of these particles. Looking closer, i.e. *fine–graining* a fluid element with density n, we have to take care of the N–particle nature and their interactions. A rough estimate that tells us in which cases we can stick to a fluid description (e.g. on cosmological scales), and in which case we need a finer description (e.g. on stellar system scales) is furnished by the following simple estimator: We consider a three–dimensional ball with radius R (a domain with volume V) and consider "particles" with a finite extension of radius r. The *collision radius*, i.e. the typical impact distance where two of the particles would collide, may be given by $2r$, and we may define an *effective cross section* roughly by $\sigma_c := 4\pi r^2$. The *mean free path* a particle can travel without collisions is therefore estimated to be $\lambda_c \cong 1/n\sigma_c$, where the product $n\sigma_c$ is called the *absorption coefficient*. Combining our formulae results in

$$\lambda_c = \frac{R^3}{3r^2 N} \quad ,$$

which we can estimate for a given system that we wish to describe.

Examples:

- for stars in a stellar accumulation we have typically $N \cong 10^5$, $R \cong 10$ pc, and $r \cong 3 \cdot 10^{-8}$ pc. We obtain $\lambda_c / R \cong 3 \cdot 10^{11}$.

- for stars in a galactic core environment we have typically $N \cong 10^6$, $R \cong 0,1$ pc, and $r \cong 3 \cdot 10^{-8}$ pc, and therefore $\lambda_c/R \cong 3 \cdot 10^6$.
- for galaxies (which are now our "particles") within a cluster environment we typically find $N \cong 10^3$, $R \cong 3$ Mpc, and $r \cong 0,01$ Mpc, and therefore $\lambda_c/R \cong 30$.

We conclude that the assumption of a *collisonless system* is very good or at least is an acceptable first approximation for most gravitational systems. Collisions, on the other hand, would – macroscopically – invoke a pressure that is relevant if we describe for example the gas component of a galaxy cluster. Here, another ingredient adds an argument for a collisionless description: it is the idea that non–baryonic *Dark Matter*, being a component with small cross–section like massive neutrinos, constitute the dominant (90%) part of the mass content of gravitational systems, in which case even baryonic gas would accurately follow structural inhomogeneities of the gravitational potential–well induced by Dark Matter. Nevertheless, we shall see that a refined description of collisionless systems in phase space will lead to an effective (in general anisotropic) pressure in space due to velocity dispersion that will be more relevant the more we go to smaller spatial scales. We also learned from the above list that it is our choice of what we consider to be the nature of our "particles", and what kind of particles consitute a macroscopic (*coarse–grained*) volume element. The precise definition of such a *continuum limit* will also be our concern in these lectures.

We shall structure the lectures such that we are commencing with cosmological scales and, by going down to smaller and smaller scales, we need to include more and more information from the detailed description of N–particle systems in phase space.