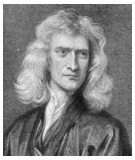
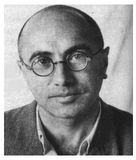



Formation Sciences de la Matière
Cours : Physique Master 2 (ENS)




Cosmologie et Systèmes Gravitationnels

TD 3

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3.1. General solution for a self–gravitating continuum of 'dust' in 1D

Solve the Euler–Newton system for plane–parallel motions (3.1),

$$\frac{d}{dt}\overline{v}_1 = g_1 \;\; ; \;\; \frac{d}{dt}\varrho = -\varrho \frac{\partial \overline{v}_1}{\partial x_1} \;\; ; \;\; \frac{\partial g_1}{\partial x_1} = -4\pi G \varrho \;\; , \tag{3.1}$$

where the cosmological constant Λ has been put to zero for simplicity.
Give the general trajectory field in terms of the Lagrangian coordinate X_1, $x_1 = f_1(X_1, t)$.
Argue, why your solution is *general*.

Hint : The Euler–Newton system is said to be *translation covariant*. This means that physical properties of the system are not affected by adding at each point a spatially constant vector. This would just imply a global shift in space without changing the spatial distribution.

3.2. Exact solution for plane–parallel motions on a Friedmann background

First, recall how we obtained the following Eulerian linear perturbation solutions in the course :

$$\mathbf{w}^\ell = \frac{2}{3t_0^2}\left(\frac{t}{t_0}\right)^{-2/3}\mathbf{A}(\mathbf{q}) + \frac{2}{3t_0^2}\left(\frac{t}{t_0}\right)^{-7/3}\mathbf{B}(\mathbf{q}) \;\; ; \tag{3.2a}$$

$$\mathbf{u}^\ell = \frac{2}{3t_0}\left(\frac{t}{t_0}\right)^{1/3}\mathbf{A}(\mathbf{q}) - \frac{1}{t_0}\left(\frac{t}{t_0}\right)^{-4/3}\mathbf{B}(\mathbf{q}) \;\; , \tag{3.2b}$$

for vector fields $\mathbf{A}$ and $\mathbf{B}$, which can be determined by the initial data. In the present example of an *Einstein–de Sitter cosmos* we have :

$$\mathbf{A} = \tfrac{3}{5}\mathbf{U}t_0 + \tfrac{9}{10}\mathbf{W}t_0^2 \quad ; \quad \mathbf{B} = -\tfrac{3}{5}\mathbf{U}t_0 + \tfrac{3}{5}\mathbf{W}t_0^2 \quad ;$$
$$\mathbf{U} := \mathbf{u}(\mathbf{q}, t_0) \ ; \ \mathbf{W} := \mathbf{w}(\mathbf{q}, t_0) \ . \tag{3.3}$$

Using the *Lagrange method*, show that – by substituting $\mathbf{q}$ with $\mathbf{X}$ – the solution for $\mathbf{u}^{\ell}(\mathbf{q}, t)$, Eq. (3.2b), is *exact* for plane–symmetric peculiar–motions on a three–dimensional Friedmann background, and that we obtain the *general solution* for the trajectory field.

Hint : start with the Euler–Newton system for the deviation dynamics given in the course,

$$\tfrac{d}{dt}\delta \ + \ \tfrac{1}{a}(1+\delta)\boldsymbol{\nabla}_{\mathbf{q}} \cdot \mathbf{u} \ = \ 0 \qquad ; \qquad \tfrac{d}{dt}\mathbf{u} + H\mathbf{u} = \ \mathbf{w} \ ;$$
$$\boldsymbol{\nabla}_{\mathbf{q}} \times \mathbf{w} \ = \ \mathbf{0} \quad ; \quad \boldsymbol{\nabla}_{\mathbf{q}} \cdot \mathbf{w} \ = \ -4\pi G a \varrho_H \delta \ , \tag{3.4}$$

and follow the line of arguments in the case of the one–dimensional solution without background, that was studied above.

Give the explicit solution of the resulting equation for the single component F_1 of the comoving trajectory field, $q_1 = F_1(X_1, t)$, by specifying the background to an *Einstein–de Sitter cosmos*.