**Horizon-less Spherically Symmetric Vacuum-Solutions in a Higgs Scalar-Tensor Theory of Gravity**

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The exact static and spherically symmetric solutions of the vacuum field equations for a Higgs Scalar-Tensor theory (HSTT) are derived in Schwarzschild coordinates. It is shown that in general there exists no Schwarzschild horizon and that the fields are only singular (as naked singularity) at the center (i.e. for the case of a point-particle). However, the Schwarzschild solution as in usual general relativity (GR) is obtained for the vanishing limit of Higgs field excitations.

PACS numbers: 14.80. Bn; 12.30Dm; 11.30 Pb.; 11.15.Kc.; 98.80.Cq.
Keywords: Higgs scalar tensor theory, gravity, Schwarzschild horizon, singularity and black holes.
The original publication is available at www.springer.com. Int.J.Theor.Phys. DOI 10.1007/s10773-007-9359-5 (2007).

The standard model (SM) of elementary particle physics has been remarkably successful in providing the astonishing synthesis of the electromagnetic, weak and strong interactions of fundamental particles in nature [1], and according to it, any massive elementary particle is surrounded by a scalar meson cloud represented by the excited scalar Higgs field which acts as a source of the mass of the particle. The associated Higgs mechanism [2, 3], therefore, provides a way of the acquisition of mass by the gauge bosons and fermions in nature. On the other hand, the gravitational interaction which completes the roster of the four fundamental interactions in nature have also seen enormous advancements and perhaps the scalar-tensor theories of gravity [4] (where the gravitational constant ought to be replaced by the average value of a scalar field coupled to the mass density of the universe) is the most natural extension of general relativity (GR) [5]. Since the long range forces (viz. the electromagnetic and gravitational interactions) are well known to be transmitted by the gravitational field and electromagnetic potential and it is therefore quite natural to suspect other long range forces by the virtue of some scalar fields (viz. Higgs scalar field). In fact, the general relativistic models with a scalar field coupled to the tensor field of GR are conformally equivalent to the multi-dimensional models [6] and using the Jordan isomorphy theorem [7] the projective spaces (like in Kaluza-Klein's theory) may be reduced to the usual Riemannian $4-dim$ spaces where a functional $5th$ component in the metric plays the role of variable gravitational constant in scalar tensor theories [9] as first predicted by Brans and Dicke [8]. In particular, utilizing the Jordan-Brans-Dicke (JBD) theory [7, 8] along with the Zee's ideas of induced gravity [10], the Higgs gravitation was first acquainted by Dehnen and Frommert [11–13] with the non-minimal coupling of the Higgs field $\phi$ to the curvature scalar $R = g^{\mu\nu}R_{\mu\nu}$ with respect to the space-time metric. The resulting Higgs scalar-tensor theory (HSTT) with subsequent developments [13–15] where the mass of the particles appear through gravitational interaction is also compatible with Dirac's large number hypothesis [16] and Einstein's Mach-Principle [17]. Moreover, based on such standpoints, the gravitational field equations with an unknown additional, ad hoc minimally coupled massless scalar field added as source in the Hilbert-Einstein field equations is also examined in detail by Hardell and Dehnen [18]. They have shown that any such scalar field influences as well as modifies the metric independently from its strength in such a way that there exists always a simultaneous solution of the massless scalar and Einstein's field equations for the static case with a scalar point-charge as a source. However, in any case no Schwarzschild horizon appears and only at the point-particle, the metric and scalar field show a singular behaviour as naked singularity which is a similar situation to that of the Reissner-Nordström solution in a more general way [19]. Furthermore, during recent years, HSTT have been extensively used to explain the various diverse physical phenomena viz. dark matter, flat rotation curves of spiral galaxies [20]-[22] and cosmological inflation [23]-[24].

In the present article, we study the vacuum solutions for the case of a non-minimally coupled Higgs field within the Higgs Scalar-Tensor theory (HSTT). It is shown that the fields are regular except for the point-particle as naked singularity and the Schwarzschild metric for the limiting case of vanishing Higgs field excitations is also derived. For this purpose, let us consider the uniquely formed Lagrangian [14] in the natural system of units as follows,

$$\mathcal{L} = \left[\frac{\breve{\alpha}}{16\pi}\,\phi^\dagger\phi R + \frac{1}{2}\phi^\dagger_{\ ;\mu}\phi^{\,;\mu} - V(\phi)\right]\sqrt{-g} + \mathcal{L}_M\sqrt{-g}, \tag{1}$$

where $\breve{\alpha}$ is a dimensionless constant and $\mathcal{L}_M\sqrt{-g}$ is the Lagrangian for the fermionic and massless bosonic fields. The Higgs potential in Eq.(1) is normalised in such a way that $V(\phi_0) = 0$ for the ground state value of $\phi$ and has

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the following form,

$$V(\phi) = \frac{\mu^2}{2}\phi^\dagger\phi + \frac{\lambda}{24}(\phi^\dagger\phi)^2 + \frac{3}{2}\frac{\mu^4}{\lambda} = \frac{\lambda}{24}\left(\phi\,\phi^\dagger + 6\,\frac{\mu^2}{\lambda}\right)^2, \tag{2}$$

with $\mu^2 < 0$ and $\lambda > 0$ as real-valued constants. The Higgs field in the spontaneously broken phase of symmetry leads to the following ground state value,

$$\phi_0\,\phi_0^\dagger = v^2 = -\frac{6\mu^2}{\lambda}, \tag{3}$$

which can further be resolved as $\phi_0 = vN$ with $N$ as a constant satisfying $N^\dagger N = 1$. With the introduction of the unitary gauge [13, 22], the general Higgs field $\phi$ may then be re-written in terms of the real-valued excited Higgs scalar field ($\xi$) in the following form,

$$\phi = v\sqrt{1+\xi}\,N\,. \tag{4}$$

The Higgs field possesses a finite range and is given by the following length scale,

$$l = \left[\frac{1+\frac{4\pi}{3\breve{\alpha}}}{16\pi G(\mu^4/\lambda)}\right]^{1/2} = M^{-1}, \tag{5}$$

where $M$ is the Higgs field mass and the gravitational coupling parameter $G$ is defined through the ground state value of the Higgs field as follows,

$$G = \frac{1}{\breve{\alpha}v^2} = -\frac{1}{\breve{\alpha}}\,\frac{\lambda}{6\mu^2}, \tag{6}$$

where the dimensionless parameter $\breve{\alpha}$ in Eq.(1) may be defined in terms of the ratio

$$\breve{\alpha} \simeq (M_P/M_W)^2 \gg 1\,. \tag{7}$$

Here $M_P$ and $M_W = \sqrt{\pi}gv$ are the Planck and gauge boson mass respectively (where $g$ denotes the coupling constant of the corresponding gauge group). However the effective gravitational coupling in terms of the Higgs field excitations is given below,

$$G_{eff} = G(\xi) = (1+\xi)^{-1}G\,. \tag{8}$$

The Eq. (8) reduces to Eq.(6) in the absence of the Higgs field excitations (i.e. $\xi = 0$) and becomes singular for a vanishing Higgs scalar field with $\xi = -1$ [22]. With such considerations and in view of the coupling (given through $\mathcal{L}_M$) of the Higgs particles to their source is only weak (i.e. $\sim G$) [14, 22, 25], the Higgs field equation takes the following form,

$$\xi^{,\mu}{}_{;\mu} + \frac{\xi}{l^2} = \frac{8\pi G}{3}T, \tag{9}$$

where $T$ is the trace of the symmetric energy-momentum tensor $T_{\mu\nu}$ belonging to $\mathcal{L}_M\sqrt{-g}$ in the Lagrangian given by Eq.(1), which satisfies the conservation law $T_\mu{}^\nu{}_{;\nu} = 0$ for the case that $\phi$ does not couple to the fermionic state $\psi$ in $\mathcal{L}_M\sqrt{-g}$. However, a coupling to SM, which means the production of the fermionic mass through the Higgs field, breaks the conservation law through a new "Higgs force" and implies simultaneously that the right hand-side of Eq.(9) vanishes identically [15, 25]. However this is a separate issue of discussion. The gravity equations of the present case which reduce to the usual ones of GR for vanishing excitations $\xi$, are then derived as follows,

$$R_{\mu\nu} - \frac{1}{2}R\,g_{\mu\nu} + (1+\xi)^{-1}\left[\left(1+\frac{3}{4}\xi\right)\frac{\xi}{l^2}\,g_{\mu\nu} + \xi_{,\mu\,;\nu}\right] = -8\pi G_{eff}\left(T_{\mu\nu} - \frac{1}{3}T\,g_{\mu\nu}\right). \tag{10}$$

It is important to notice that in view of the structure of $l$ in the HSTT, only large values of the length scale $l$ are expected. Indeed, only such values within the HSTT lead to the correct explanation of the solar-relativistic effects of GR [26] as well as of flattened rotation curves of spiral galaxies without assuming dark matter [20, 22]. It is important to notice that the limiting case of a vanishing Higgs field mass (i.e. $l \to \infty$ ) can be understood as a double limit $\mu^2 \to 0$ and $\lambda \to 0$, so that $\mu^4/\lambda = 0$ and $v^2 = \mu^2/\lambda$ which is finite quantity and remain valid throughout. Thus, the ground state value keeps the degeneracy and the symmetry remains broken at low energies. The scalar field still changes the usual dynamics after symmetry breakdown and the excitations are in general non-vanishing. A detailed analysis in the limit of vanishing non-minimally coupled Higgs field masses is therefore important to give general characteristics of the dynamics within the HSTT (especially if these masses are expected as small). As such, in order to solve the Eq.(9) with a vanishing Higgs field mass, let us consider the following line element in the spherical symmetry,

$$ds^2 = e^\nu (dt)^2 - e^\lambda dr^2 - r^2\,d\Omega^2\,, \tag{11}$$

where $\nu$ and $\lambda$ are the functions of $r$ alone and $d\Omega^2 = (d\vartheta^2 + \sin^2\vartheta\,d\varphi^2)$ is the metric on a $2-dim$ unit sphere. Now with the point-mass at $r = 0$, the Higgs field equation given by Eq.(9) takes the form given below,

$$\xi'' - \frac{1}{2}(\lambda' - \nu')\,\xi' + \frac{2}{r}\xi' = 0\,, \tag{12}$$

where the prime denotes the differentiation with respect to $r$. The first derivative of the excited scalar field $\xi$ from Eq.(12) in the case of a point-mass (with internal structure (pressure)) at $r = 0$ then reads,

$$\xi' = \frac{A}{r^2}\, e^{w/2} = \frac{A}{r^2}\, e^{(\lambda - u/2)}\,, \tag{13}$$

where $u = \lambda + \nu$ and $w = \lambda - \nu$. However, the integration constant $A$ is given below according to Eq.(9) in the limit $r \to \infty$,

$$A = -\frac{2}{3}\, G \int T\sqrt{-g}\, d^3x\,. \tag{14}$$

The non-trivial field equations of gravity associated to (10) then acquires the following form for the case of a point-mass in vacuum,

$$\frac{1}{2} r w' = 1 - e^{(u+w)/2} + r\, q'\,, \tag{15}$$

$$u'\,(1 + \frac{r}{2} q') = \frac{r}{2} q'\,(w' - \frac{4}{r})\,, \tag{16}$$

$$\frac{1}{2}\,(u' - w') = \frac{B}{r^2}\, e^{w/2 - q} = \frac{B}{A}\, q'\,, \tag{17}$$

where $q' = \xi'(1+\xi)^{-1}$ and $B$ is an integration constant. Using the value of $u'$ given in Eq.(16), the Eq.(17) leads to the following decoupled equation,

$$w' = -\frac{2(A+B)}{r^2}\, e^{w/2 - q} - \frac{AB}{r^3}\, e^{w - 2q}. \tag{18}$$

Now, using the Eqs.(15) and (18) one can also immediately deduce the identity given below,

$$(e^{q + u/2} - e^{q - w/2}) = \frac{(2A+B)}{r} + \frac{AB}{2r^2}\, e^{w/2 - q}\,, \tag{19}$$

and, therefore, only the differential Eq.(18) remains to be solved. These considerations further lead a solution of the excited Higgs field given by Eq.(13) in the following form for $B \neq 0$,

$$\xi = e^q - 1 = e^{\frac{A}{2B}(u - w)} - 1\,. \tag{20}$$

The Eq.(20) clearly indicates that such excitations of the Higgs scalar field are only possible for a non-vanishing value of the integration constant $A$ given by Eq.(14). The exponential term with the coefficients of amplitude and gravitational potential gives the deflection from completely vanishing scalar fields. However, in order to determine the meaning of the integration constant $B$ we consider the asymptotic case $r \to \infty$ of the potentials( i.e. $|w| \ll 1$, $|u| \ll 1$) which in turn results,

$$u = 2\frac{A}{r} + \frac{AB}{2r^2}\,, \tag{21}$$

$$w = \frac{2(A+B)}{r} + \frac{AB}{2r^2}\,, \tag{22}$$

and consequently,

$$\nu = \frac{(u - w)}{2} = -\frac{B}{r}\,, \tag{23}$$

$$\lambda = \frac{(u + w)}{2} = \frac{AB}{2r^2} + \frac{(2A+B)}{r}\,, \tag{24}$$

which defines the integration constant $B$ in the asymptotic limit as follows,

$$B = \frac{2\tilde{M}_S}{\breve{\alpha} v^2} = 2\tilde{M}_S G\,. \tag{25}$$

The equation (25) is valid in view of the equation of motion of the line element (11) where $\tilde{M}_S$ is the asymptotically visible mass of the particle which represents the Schwarzschild mass. Further, the differential Eq.(18) is an Abelian one and can be solved exactly by making the following substitution,

$$e^{w/2 - q} = r\,\tilde{g}(r) = r\,\tilde{g}\,. \tag{26}$$

The Eq.(18) then acquires a much simpler form as given below,

$$r\tilde{g}' = \alpha\tilde{g}^3 - K\,\tilde{g}^2 - \tilde{g}\,, \tag{27}$$

where $K = 2A + B$ and $\alpha = -\frac{AB}{2}$. The Eq.(27) can now be integrated by using the method of separation of variables, which for $\alpha \neq 0$ reduces to the following form,

$$\left|\frac{\tilde{g}^2}{1 + K\tilde{g} - \alpha\tilde{g}^2}\right| \left|\frac{\sqrt{K^2 + 4\alpha} + K - 2\alpha\tilde{g}}{\sqrt{K^2 + 4\alpha} - K + 2\alpha\tilde{g}}\right|^{\frac{K}{\sqrt{K^2+4\alpha}}} = \frac{C}{r^2}\,. \tag{28}$$

The integration constant $C$ in Eq.(28) can be calculated in the Minkowskian limit [18] and is given as,

$$C = \left(\frac{\sqrt{K^2 + 4\alpha} + K}{\sqrt{K^2 + 4\alpha} - K}\right)^{\frac{K}{\sqrt{K^2+4\alpha}}}\,. \tag{29}$$

Here the constant $K$ turns out to be a generalised mass parameter and $\alpha$ itself can be interpreted as a product-charge. Thus, the non-minimally coupled massless Higgs field within the HSTT acts in an analogous way to a massless scalar field within the Einstein's theory of gravity [18]. Moreover, the symmetry breakdown is still intact since the ground state stays degenerate and doesn't switch over to the Wigner mode. In view of the Eqs.(19), (20) and (26), the metric components given by Eq.(11) and the scalar field by the Eq.(13) for the case $B \neq 0$ may then be expressed in terms of $\tilde{g}$ in the following form,

$$e^{\nu} = \left[\frac{1}{r^2\tilde{g}^2}\,(1 + K\tilde{g} - \alpha\tilde{g}^2)\right]^{\frac{B}{K}}\,, \tag{30}$$

$$e^{\lambda} = 1 + K\tilde{g} - \alpha\tilde{g}^2\,, \tag{31}$$

$$\xi = -1 + \left[\frac{1}{r^2\tilde{g}^2}(1 + K\tilde{g} - \alpha\tilde{g}^2)\right]^{\frac{A}{K}}\,. \tag{32}$$

The only effective physical parameters remaining in the theory of the present model are only the integration constants $A$ and $B$ which are defined by the Eqs.(14) and (25), respectively. Unfortunately, it is quite difficult to solve the equation (28) for $\tilde{g}$ explicitly while it is exactly solvable for the limiting case $A = 0$ (i.e. for the equation of state $\varrho - 3p = 0$) with $B \neq 0$ in the following form,

$$\tilde{g} = \frac{1}{r}\left(1 - \frac{B}{r}\right)^{-1}, \tag{33}$$

and the Eqs.(30) and (31), in turn, results,

$$e^{\nu} = e^{-\lambda} = \left(1 - \frac{B}{r}\right). \tag{34}$$

The equation (34) indicates that the metric components of line element given by equation (11) correspond to the usual Schwarzschild metric (with associated features) which appears in this form only for the limiting case of the vanishing Higgs scalar field excitations (i.e. $\xi = 0$)[27]. However, for the general values of $A$, the qualitative results shown in the work of Hardell and Dehnen [18] are valid. It is worth mentioning that the higher values of $A$ (30) lead the decrease in $\nu$ through the exponent $B/K$. In fact, the metric and scalar field are regular everywhere with exception of $r = 0$ as naked singularity and there exists no Schwarzschild horizon except for the case of vanishing scalar field excitations. Therefore, Black holes (in the usual sense) do not appear for the case $A \neq 0$. However, it still remains to see the exact influential role of the Higgs scalar field excitations on the system in view of the different non-vanishing values of $A$ and may be helpful in explaining the quintessence- and dark energy- oriented problems which shall be dealt in our forthcoming communications.

**Acknowledgments**

The authors NMBR and HN are thankful to Prof. H. Dehnen and Prof. P. K. Raina for the hospitality during their stay at the Fachbereich Physik, Universität Konstanz and Centre for Theoretical Studies, Indian Institute of Technology, Kharagpur, India respectively. The authors would also like to thank Prof. F. Steiner and Dr. R. Aurich for stimulating discussions and various useful comments.

[1] M. E. Peskin and D. V. Schroeder, An Introduction to Quantum Field Throry, Addison Wesley Reading, Massachusetts (1995).
[2] P. W. Higgs, Phys. Rev. Lett. 12 (1964) 132.
[3] F. Engelert and R. Brout, Phys. Rev. Lett. 13 (1964) 321.
[4] Y. Fujii and K. Maeda, The Scalar-Tensor Theory of Gravitation, Cambridge University Press, Cambridge (2003).
[5] S. Weinberg, Gravitation and Cosmology : Principles and Applications of General Theory of relativity, Cambridge University Press, Jhon Wiley and Sons Inc. (1972).
[6] S. Cotsakis, gr-qc79712046.
[7] P. Jordan, Schwerkraft und Weltall, Vieweg (Braunschweig) (1955).
[8] C. H. Brans and R. H. Dicke, Phys. Rev. 124 (1961) 925.
[9] B. Fauser, Gen. Rel.and Grav.,33(2001) 875.
[10] A. Zee, Phys. Rev. Lett. 42 (7) (1979) 417.
[11] H. Dehnen and H. Frommert, Int. J. Theor. Phys. 29(4) (1990) 361.
[12] H. Dehnen and H. Frommert, Int. J. Theor. Phys. 29(6) (1990) 537.
[13] H. Dehnen and H. Frommert, Int. J. Theor. Phys. 30 (1991) 985.
[14] H. Dehnen, H. Frommert and F. Ghaboussi, Int. J. Theor. Phys. 31 (1) (1992) 109.
[15] H. Dehnen, H. Frommert, Int. J. Theor. Phys. 32 (7) (1993) 1135.
[16] C. H. Brans, "Gravity and the Tenacious Scalar Field", gr-qc/9705069 (1997).
[17] A. Einstein, Phys. Zs. 14 (1913) 1260.
[18] A. Hardell and H. Dehnen, Gen. Rel. and Grav. 25 (11) (1993) 1165.
[19] H. Reissner, Ann. der Physik 50 (1916) 106.
[20] E. Gessner, Astrophys. and Sp. Sc. 196 (1992) 29.
[21] M. A. Rodríguez-Meza and J. L. Cervantes-Cota, Mon. Not. R. Astron. 350 (2004) 671.
[22] N. M. Bezares-Roder and H. Dehnen, "Higgs Scalar-Tensor Theory for Gravity and the Flat Rotation Curves of Spiral Galaxies", to appear in Gen. Rel. and Grav. (2007).
[23] J. L. Cervantes-Cota and H. Dehnen, Phys. Rev. D 51 (1995) 395; Nucl. Phys. B 442 (1995) 391.
[24] A. L. Berkin and K. Maeda, Phys. Rev. D 44 (1991) 1691.
[25] N. M. Bezares Roder and H. Nandan, "Spontaneous Symmetry Breakdown and Perspective of Higgs Mechanism", hep-ph/0603168 (2006).
[26] M. Kessler, Solarrelativistische Effekte in einer Higgsfeld-Skalar-Tensor-Theorie, Universität Konstanz, Konstanz (1997).
[27] N. M. Bezares Roder, H. Nandan and H. Dehnen, "Schwarzschild solution in a Higgs Scalar-Tensor Theory of Gravity", work presented in the 24th IAGRG meeting (2007), New Delhi, India on the Recent advances in Gravitation and Cosmology.