

# Spontaneous Symmetry Breakdown and Critical Perspectives of Higgs Mechanism

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The foundations of the mass generation mechanism of particles are reviewed. The Spontaneous Symmetry Breaking (SSB) process within the standard model (SM) and the minimal supersymmetric standard model (MSSM) is used to explore the present status of the Higgs Mechanism along with the constraints in detecting the Higgs particles in experiments. The possible explanations and generalizations for the case that the Higgs particles should not appear or to couple the Higgs Mechanism (because of their gravitational nature of interaction) are also discussed in detail in view of the Higgs field gravity.

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## I. INTRODUCTION

The mass of any physical object is thought to be an entity which quantifies its amount of matter and energy. However, conceptually, it is defined as inertial and gravitational (passive and active) mass. The inertial mass is indeed a measure of an object's resistance to the change of its position due to an applied force. On the other hand, the passive gravitational mass measures the strength of an object's interaction with the gravitational field, while the active one is a measure of the strength of the gravitational field due to a particular object. However, Einstein's principle of equivalence asserts the equality of the gravitational ( $m_g$ ) and inertial mass ( $m_i$ ) of a body. In fact, by now all the experiments have failed to make any difference between them [1, 2]. More generally, such equivalence between the inertial and gravitational mass is included, for instance, in the weak equivalence principle (WEP), which confirms the universality of free fall such that all bodies in a given gravitational field and at the same space-time point would undergo the same acceleration. Further, the strong equivalence principle (SEP) is a generalization in the sense that it governs all the effects of the gravitational interaction on all physical systems and holds for all the *laws of nature*. In fact, the WEP replaces the *laws of nature* (which might be the case for SEP) by the *laws of motion of freely falling bodies* [1]. Independently of the equivalence of masses, however, the origin of mass itself is another conceptual problem of physics to solve, and the mechanism through

which particles acquire mass is indeed an important subject from the point of view of the basic constituents and interactions among them in nature. In this context, the mass generation is identified with the symmetry breakdown of the Lagrangian corresponding to a particular theory where the mass comes as a consequence of the symmetry loss and features of the self-interactions in the standard model (SM), which is perhaps the most celebrated theory of modern elementary particle physics. The SM is a full relativistic quantum field theory, and it has been indeed incredibly successful in describing the electromagnetic, weak and strong interactions between the basic constituents (quarks and leptons) with the symmetry group  $G_{SM} \equiv SU(3)_C \otimes SU(2)_W \otimes U(1)_Y$  down to the distances as small as  $10^{-16} cm$ . In SM, the interaction of the constituents of matter with the Higgs field allows all the particles to have different mass [3]. The mass of the particles obtained via the so-called Higgs Mechanism is then proportional to the vacuum expectation value (VEV) of the Higgs field so that mass can be given in terms of the parameters of the Higgs potential. However, in GR it is still not possible to explain the origin of mass from the curved space-time, and the mass is used as a parameter with the equivalence  $m_g \equiv m_i \equiv M$ . The SM, therefore, provides a unique way to explain the acquisition of mass by basic constituents of matter via Higgs Mechanism [4]. Moreover, the cosmological consequences of mass production still demands further explanation and a unified theory in this context is still lacking. However, some physical aspects to completely solve the issue within gauge-gravitation theories, Supersymmetry (SuSy), Supergravity (SuGra) and Loop Quantum Gravity (LQG) are really promising. It is noteworthy that the appearance of mass in SM by the virtue of Higgs Mechanism comes in a natural way as well as it co-exists peacefully in various known processes of the physical world as predicted by SM itself. Unfortunately, some of the basic aspects of the Higgs Mechanism are still unknown as the Higgs particle is still not an experimental reality. However, with the developments in SuSy (which emphasize a symmetry between fermions and bosons), it leads to the

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\*\* This article is dedicated to Professor Heinz Dehnen on his 70th birth anniversary for his pioneering contribution to the Higgs field gravity.

possibility to cancel unphysical quadratic divergences in the theory as well as it provides an answer to the hierarchy problem between the electro-weak (EW) ( $\sim 10^2\text{GeV}$ ) and Planck ( $\sim 10^{19}\text{GeV}$ ) scale [3]. Therefore, the supersymmetric version of the SM may play an important role to stabilize the hierarchy against quantum corrections, and in the minimal supersymmetric SM (MSSM) with the radiative EW symmetry breaking, the stability of the Higgs field leads to mass generation to be around the EW scale. Remarkably enough, the problem of mass generation and its explanation is still a very important subject which needs to be explored in view of the various developments in modern physics, and it is certainly not a closed chapter for further discussions. In fact, the Higgs Mechanism along with the search of the Higgs particles at higher and higher energies has narrowed down the scope for other theories in this regard and has become a natural tendency to find an appropriate answer to understand the mass generation [3].

In the present article, the problems associated with the mass generation are revisited from the perspectives of the different well known mass-containing Lagrangians. The Higgs Mechanism in view of the SM is summarized with the current experimental status. The various phenomenological aspects related to the Higgs Mechanism (in view of the different unification schemes of the fundamental interactions) are reviewed. The gravitational-like interactions and the possibility without interacting Higgs particles, which puts some constraints on Higgs Mechanism, are also discussed by the virtue of the Higgs field gravity. The impact of the Higgs scenario on the physical world is concluded along with its possible future prospects.

## II. THE MASS GENERATION AND DIFFERENT SYMMETRY BREAKING MODES

In order to have a discussion on the mass generation mechanism within the notions of the analytical mechanics, let us first write Hamilton's principle of the stationarity (or least) action in the following form,

$$\delta \int \mathcal{L} \sqrt{-g} d^4x \equiv 0. \quad (1)$$

In fact, there are two possible ways to introduce the mass term in a particular theory:

(i) With an additional term ( $\mathcal{L}_m$ ) containing the mass in the general Lagrangian  $\mathcal{L}$ .

(ii) With the SSB via an extra term  $\mathcal{L}_H \equiv \mathcal{L}_5$  having a 5<sup>th</sup> symmetry-breaking force.

It is possible to achieve the well-known equations (viz Schrödinger, Klein-Gordon and Dirac equation) in non-relativistic as well as in relativistic quantum mechanics (RQM) with the aforesaid first choice. The Lagrangians from which these equations can be derived, actually contain the mass terms and have the following form in the

natural system of units,

$$\mathcal{L}_{SE} = -\frac{1}{2M} \psi_{,k}^* \psi_{,k} + \frac{i}{2} (\psi^* \psi_{,t} - \psi \psi_{,t}^*) - V(\psi \psi^*), \quad (2)$$

$$\mathcal{L}_{KG} = \frac{1}{2} (\psi_{,\mu}^* \psi^{,\mu} - M^2 \psi^* \psi), \quad (3)$$

$$\mathcal{L}_D = \frac{i}{2} (\bar{\psi} \gamma^\mu \psi_{,\mu} - \bar{\psi}_{,\mu} \gamma^\mu \psi) - M \bar{\psi} \psi, \quad (4)$$

where  $M$  denotes the mass and  $_{,\mu} \equiv \partial_\mu$ , while  $\gamma^\mu$  represent the well-known Dirac matrices. The problem with this choice, however, relies on the well tested fact of parity violation (viz the CP violation in the  $\beta$ -decay in Wu-like experiments in the electro-weak interactions). This violation cannot be achieved by adding mass by hand because in the equations (2-4), the left- and right-handed particles couple all in the same way to vector-bosons in order to preserve the gauge invariance [3]. Moreover, a massive propagator (which gives the probability amplitude for a particle to travel from one place to another in a given time, or to travel with a certain energy and momentum, in this case for massive virtual particles) does not lose its longitudinal term and as well does not transform in a (transversal), massless, one in the limit  $M \rightarrow 0$ . As a consequence of this, most closed Feynman graphs diverge, which makes the theory non-renormalizable. It is therefore needed to have a theory with the requirements of renormalizability and which can be achieved by the spontaneous symmetry breakdown, for which the existence of an extra scalar field (the Higgs field) is needed to make the theory (e.g. SM) mathematically consistent [3, 5]. The characteristics of different symmetry-breaking modes may therefore be defined from the point of view of the parity violation and renormalization. For the parity violation and well behaved propagators, the best choice is the symmetry breaking where the mass is produced as a consequence of the *loss* of symmetry and self-interactions. The requirement for such breakdown of symmetry also demands another gauge invariant (i.e. current-conserving) term in the Lagrangian, which is identified with the interaction of the spontaneous production of mass. The symmetry breakdown can be given through different modes, depending on the properties of the ground state. In different quantum field theories, the ground state is the vacuum state, and it is therefore important to check the response of the vacuum state to the symmetry breaking. As such, there are three main modes [6] as given below:

- (i) The *Wigner-Weyl* mode,
- (ii) The *Nambu-Goldstone* mode,
- (iii) The Higgs mode.

In these processes, the symmetry group  $G$  breaks down to a rest-symmetry group  $\tilde{G}$  (i.e.  $G \rightarrow \tilde{G}$ ) with  $\tilde{G} = \bigcap_{r=1}^n \tilde{G}_r$  where  $n > 1$  is valid for more than one breaking process. For instance, in the SM, the breaking

$SU(3)_C \otimes SU(2)_W \otimes U(1)_Y \rightarrow SU(3)_C \otimes U(1)_{em}$  is valid, while for the grand unified theory (GUT) under  $SU(5)$ , the breaking  $SU(5) \rightarrow SU(3)_C \otimes SU(2)_W \otimes U(1)_Y$  also takes place at about  $10^{15} GeV$ . However, another interesting example of symmetry-breaking comes from the fundamental asymmetry between space and time which is found in the signature of the relativistic metric [7] and is mostly given in *ad hoc* manner. Such asymmetry may be generated as a property of the ground state following a symmetry breakdown in the universe from which the structure of the quantum field theory and gravitational field equations would be derivable.

In particular, the *Wigner*-Weyl mode is the most usual symmetry-breaking mode in quantum mechanics (QM), with a real invariant vacuum which can be identified with the classical one as follows,

$$U|0\rangle = |0\rangle. \quad (5)$$

The *Wigner*-Weyl mode is indeed related to the existence of degeneracy among particles in the multiplets and the violation of which enforces an explicit symmetry breakdown in the Hamiltonian  $H$ . Such situation appears in the Zeeman effect where turning-on of the external fields causes the breakdown of the rotational symmetry. One more example of the *Wigner*-Weyl mechanism may be seen in the breaking of  $SU(3)_C$  to  $SU(2)_W$  due to the effect of hypercharges, and which further breaks in  $U(1)$  because of Coulomb interaction. However, the  $U(1)$ -symmetry remains unbroken because of the current-conservation law [6]. Further, in both the Nambu-*Goldstone* and Higgs modes, the symmetry is actually not lost but camouflaged and hidden in the background of the mass generation. However, these two modes differ from each other through their gauge-symmetry while both of them are given by the vacuum defined as follows,

$$U|0\rangle \neq |0\rangle. \quad (6)$$

It is worth mentioning that the Nambu-*Goldstone* mode works *globally* while the Higgs mode acts *locally* in view of gauge invariance. The main difference between them is that in the Nambu-*Goldstone* Mechanism both massive (Higgs) and massless (Goldstone) particles (generally bosons) appear, while in the Higgs Mechanism only the massive particles are present and the mass acquisition of gauge bosons is at the cost of the Goldstone particles, which are gauged away unitarily. The degrees of freedom of the massless particles, however, do not disappear from the physical spectrum of the theory. In general sense, the gauge fields absorb the Goldstone bosons and become massive while the Goldstone bosons themselves become the third state of polarization for massive vector-bosons. This interpretation is analogous to the *Gupta-Bleuler* Mechanism where, with the quantization of a massless field  $A^\mu$ , the temporal degree of freedom of  $A^\mu$  (i.e.  $A^0$ ) cancels with longitudinal space-like components of  $p_\mu \cdot \mathbf{A}$  in a way that  $A_\mu$  becomes the transversal components of  $\mathbf{p} \times \mathbf{A}$  [6]. In the Higgs Mechanism,

the Goldstone mode cancels the time-like components of the gauge fields in such a way that the three space-like components remain intact and  $A_\mu$  behaves like a massive vector-boson. The mass generation by this sort of way can best be identified in the Meissner effect in conventional superconductivity, and the SSB may therefore be applicable in explaining such mechanism also in non-relativistic theories [8]. However, an analogy between the Higgs Mechanism and the Meissner effect may be explained in terms of the Yukawa-Wick interpretation of the Higgs Mechanism where the Goldstone bosons at unitary gauge vanish because of the existence of long-ranged forces while their short-ranged behavior may be transcribed by Yukawa's theory for the massive fields. The condensed electron-pairs (the Cooper pairs) in the ground state of a superconductor may then be identified with a Higgs field which leads the magnetic flux expulsion with a finite range given by the penetration depth, which is basically the reciprocal effective mass acquired by the photons. For instance, in a Scalar-Tensor Theory (STT) of gravitation with symmetry breaking, which is derivable as the simplest Higgs-curvature coupled theory [9, 10] and is based on the analogous properties of the Higgs and gravitation [11, 12], the Higgs field of the theory has a finite range which is the inverse of its Higgs field mass [13]. There, analogies between the Higgs field for the Schwarzschild metric and the London equations for the Meissner effect also exist. In general, the coupling between the superconductor and Higgs is of more profound nature since it helps in modern contributions to understand SM, especially in context of dual QCD where the Higgs field with magnetic charge leads to the Meissner effect of color electric flux which provides a unique way to understand the quark confinement mechanism in the background of the magnetic charge condensation [14, 15].

### III. THE HIGGS MECHANISM AND UNITARY GAUGE

The mass generation through an interaction with a non-empty vacuum can be traced back to the  $\sigma$ -model proposed by Schwinger where the  $\sigma$  and  $\varphi_i$  ( $i = 1, \dots, 3$ ) lead to the appearance of three massive and one massless vector bosons. The  $\sigma$ -model seems typical in view of the physical economy in comparison to the Higgs Mechanism, which demands the appearance of only one scalar field  $\phi$  [16]. In fact, the scalar multiplet in the SM belongs to a doublet representation of the gauge group in the following form,

$$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}, \quad (7)$$

which is defined with a non-trivial vacuum state having the characteristics of symmetry breaking of the gauge group  $G$  to the rest-symmetry of the isotropy group  $\hat{G}$ . The complex field  $\phi^0$  can be further re-written in terms

of real fields (i.e.  $\phi^0 = (\tilde{\sigma} + i\chi)/\sqrt{2}$ ). With the spontaneous breakdown of the gauge symmetry, the minimal value of the energy-density  $u$  is taken by the ground state value  $\phi_0 = v$  with  $\langle \tilde{\sigma} \rangle = v$ . The  $\tilde{\sigma}$  and  $\chi$ -fields may then be identified with the Higgs particles and Goldstone bosons respectively. The symmetry of the Lagrangian is then broken when particles fall from their false vacuum (with  $\phi = 0$ ) to the real one ( $\phi = v$ ). In general, for such SSB, the less energy is required to generate a new particle (i.e. Higgs particle) with the associated features of the self-interaction. With Higgs bosons as neutral particles, the photons are not able to *see* them and remain massless in electrodynamics, while, however, the neutral  $Z$ -bosons couple to Higgs bosons via a Weinberg-mixture with charged  $W$ -boson fields.

The Higgs mode, in fact, does not need to violate parity, while this indeed occurs in the  $\sigma$ -model [3]. Nevertheless, this violation is given in SM through the isospin scalar field  $\phi$  as a doublet in its iso-vectorial form instead of only an iso-scalar with  $\phi = vN$ , where the unitary vector  $N$  in the isospin-space satisfies  $N^\dagger N = 1$ . On the other hand, the right-handed bosonic multiplets are only iso-scalar, while left-handed ones are iso-doublets (for up and down states). The multiplets acquire mass through the components of  $N$ , which in the unification model (viz  $SU(5)$  for instance) is matrix-valued for the first symmetry-breaking. The mass of the states is determined by the VEV  $v$  and an arbitrary coupling constant  $g$ . However, the parity violation appears naturally through the gauging of the group with the help of the non-canonical Pauli- $\sigma$ -operators [17, 18]. Moreover, in the LQG, such parity violation is described by matching the *immirzi* parameter (which measures the size of the quantum of area in Planck units) with the black hole entropy [19].

In general, the simplest way to generate the spontaneous breakdown of symmetry is to have a Lagrangian with the Higgs potential  $V(\phi)$  and a transformed gauged field  $\phi'_a = U\phi_a = e^{\tilde{\lambda}^a \tau_a} = e^{\chi_a}$  in the following form,

$$\mathcal{L}_H = \mathcal{L}(\phi) = \frac{1}{2} \phi_{;\nu}^\dagger \phi^{;\nu} - V(\phi) = \mathcal{L}(\phi'), \quad (8)$$

where  $;\mu \equiv D_\mu$  is the covariant derivative and the potential  $V(\phi)$  has the form as follows,

$$V(\phi) = \frac{\mu^2}{2} \phi^\dagger \phi + \frac{\lambda}{4!} (\phi^\dagger \phi)^2, \quad (9)$$

where  $\mu^2 < 0$  and  $\lambda > 0$ . Such theories are called  $\phi^4$ -theories. The last term in the potential (9) is not bilinear and it is crucial for the apparent symmetry breakdown. The Lagrangian given by equation (8) is invariant under the spatial-inversion (i.e.  $\phi \rightarrow -\phi$ ) with the features of the tachyonic condensation (i.e. condensate for an imaginary mass with  $\mu^2 < 0$ ). Such conditions are needed to stay with the Higgs mode, which otherwise be-

comes a Wigner mode with classical vacuum where self-interactions lack to produce the necessary Higgs Mechanism at the relatively low energies of the hodiernal universe. However, with the possibility of the tachyonic condensation, the ground state  $\phi_0$  becomes twice degenerate and  $\phi_z = 0$  has a maximal value for the energy-density  $u$ . The minimum energy is then given by the non-vanishing Higgs ground state value (i.e.  $v \neq 0$ ) in the following form:

$$u_0 = u(\phi_0) = -\frac{3}{2} \frac{\mu^4}{\lambda} \equiv u_{min},$$

$$\phi_0^{(+)} = \sqrt{-\frac{6\mu^2}{\lambda}} e^{i\alpha} \equiv \tilde{v} = v e^{i\alpha} \neq 0. \quad (10)$$

For a pure scalar case,  $v$  is to be chosen between  $\phi_0^{(-)}$  and  $\phi_0^{(+)}$ . As such, in technical jargons, the circle of the localized minima for the minimality condition of  $u$  is popularly known as a *Mexican hat* and the regions with different  $\phi_0$ -values are called the *topological defects* while those changing with the values  $\phi = v \leftrightarrow -v$  are termed as *interface domains*. Further, it is also possible to have the choice of  $\alpha = 0$  without making any restriction to the system since this does not demand any kind of physical changes. However, this choice does not allow mass to go through the phase transitions without changing its vacuum value. Therefore, even if the Lagrangian is invariant under phase transitions, it must suffer the loss of invariance explicitly through its ground state, and the particles that fall in this state interact with the Higgs bosons and slow down. In particular, in view of the Special Relativity (SR), the massless particles travel with the speed of light  $c$  and massive ones have as speed  $v < c$ . So the mass generation of the particles may be interpreted in relation to their interaction with the Higgs field. In fact, the energy of the system is nothing but a meager and  $\phi$  lies near the minimum of energy. It is, therefore, possible to expand the scalar field around its minimal state with its excited values  $\hat{\phi}$  in the following form:

$$\phi = v + \hat{\phi}. \quad (11)$$

The Lagrangian (8) may now be given in iso-scalar form (only up to second order terms) as follows,

$$\mathcal{L}(\hat{\phi}) = \frac{1}{2} \hat{\phi}_{;\nu}^\dagger \hat{\phi}^{;\nu} - \frac{M_H^2}{2} \hat{\phi}^2 - \frac{\lambda}{3!} v \hat{\phi}^3 - \frac{\lambda}{4!} \hat{\phi}^4 \neq \mathcal{L}(-\hat{\phi}). \quad (12)$$

The first term in the Lagrangian (12) corresponds to the kinetic energy of the Higgs field while the second one represents the mass term (i.e.  $M_H^2 \equiv -2\mu^2$ ) for the Higgs field. In fact, due to the presence of the term for the excited field (i.e.  $\hat{\phi}^3$ ) in the Lagrangian (12), the symmetry is suddenly broken since the Lagrangian (12) is not spatially invariant anymore. However, the Lagrangian in the iso-vectorial form may be re-written as

$$\mathcal{L}(\hat{\phi}) = \frac{1}{2} \hat{\phi}^{\dagger a} \cdot_{,\nu} \hat{\phi}_a \cdot_{,\nu} + \frac{1}{2} g^2 A_{\mu a}{}^b \phi_0^{\dagger a} A^\mu{}_{b c} \phi_{0c} - \frac{\lambda}{4!} (\phi_0^{\dagger a} \hat{\phi}_a + \hat{\phi}^{\dagger a} \phi_{0a})^2, \quad (13)$$

where  $\phi_a$  is a scalar quantity in spin-space. The Lagrangian in iso-vectorial form is invariant under local transformations for which it is needed to define the covariant derivative (different for left- and right-handed states because of the couplings which give the parity violation). As such, the massive term of the excited Lagrangian with these excited Higgs scalar field leads to the term of the mass of the gauge bosons as

$$M_A = \frac{1}{2} g^2 A_{\mu a}{}^b \phi_0^{\dagger a} A^\mu{}_{b c} \phi_{0c} \sim (\mathcal{M}^2)^{ij} A_{\mu i} A^\mu{}_j, \quad (14)$$

where the mass term for the gauge boson  $A_\mu$  comes from the covariant derivative, and the mass-square matrix (operator), which is symmetric and real, is given below in the natural system of units:

$$(\mathcal{M}^2)^{ij} = 4\pi g^2 \phi_0^\dagger \tau^{(i} \tau^{j)} \phi_0 = 2\pi g^2 v^2 (c^{ij} \mathbf{1} + N^\dagger d^{ij}{}_k \tau^k N). \quad (15)$$

Such broken phase of symmetry cannot be reached by perturbative expansion techniques from the normal vacuum. The SSB may therefore be thought of as a phase transition which is manifestly non-perturbative. However, the Higgs field does not give mass to the neutrinos in the usual form of SM. In the equation (15),  $\tau^i = \tau^{i\dagger}$  are the generators of the gauge group and they satisfy the following properties for the rest and broken phase of the symmetry respectively:

$$\left. \begin{array}{l} \tau_a^j{}^b N_b = 0 \\ \tau_a^j{}^b N_b \neq 0 \end{array} \right\}. \quad (16)$$

The diagonal components of the mass-square matrix are positive definite and correspond to the mass of the gauge bosons coupled to the scalar vector field  $\phi$ . The masses corresponding to the Higgs scalar multiplet are then  $M_H^2 = -2\mu^2$  and  $M_G = 0$  where the massless Goldstone boson ( $M_G$ ) belongs to the Nambu-Goldstone mode because of the global symmetry breakdown which carries the quantum number of the broken generator. Further, with the conditions of conserved current corresponding to an exact symmetry of the Lagrangian, the non-invariant vacuum follows  $\chi_a|0\rangle \neq 0$  (where  $\chi_a$  is the Goldstone boson field) while Lorenz invariance implies that  $\tilde{\lambda}^a \tau_a = \chi_a$  is valid at least for one  $a$ . Moreover, there must be a state  $|m\rangle \in \mathcal{H}$  with  $\langle m|\chi_a|0\rangle \neq 0$  for a massless spin-0 particle. Nevertheless, in nature such massless scalar spin-0 particles do not seem to exist and the Goldstone bosons would give rise to long-ranged forces in classical physics to generate new effects in various scattering and decay processes in nature. The possible non-relativistic long-range forces arising from the existence of massless Goldstone particles are spin-dependent, and it is quite difficult

to observe them directly. However, in principle, the  $\gamma_5$ -couplings along with the CP-violation would change to scalar interactions which may then subsequently lead to spin-independent long-range forces [20]. The existence of such Goldstone bosons may also affect the astrophysical considerations with some sort of new mechanism for the energy loss in stars. Furthermore, the excited Higgs field distinguishes from the ground state by a local transformation that can be gauged away through an inverse unitary transformation  $U^{-1}$ . Such unitary transformations contain the Goldstone fields  $\tilde{\lambda}$  (as the generator of symmetry) in the following form,

$$U = e^{i\tilde{\lambda}^a \tau_a} = e^{i\chi_a}, \quad (17)$$

and it is possible to gauge out the Goldstone bosons from the theory by the following unitary gauge transformations,

$$\phi = \frac{\rho}{v} U \phi_0 = \rho U N, \quad (18)$$

and consequently we have,

$$\phi \rightarrow U^{-1} \phi = \rho (U^{-1} U) N = \rho N; \quad \psi \rightarrow U^{-1} \psi, \quad (19)$$

with  $\rho = \phi^\dagger \phi$ ,  $\phi = v(1 + \varphi) = v\zeta$ . The absence of the Goldstone bosons is then mathematically permitted, which indicates that Goldstone's rule of massless particles in the broken phase of symmetry is only valid for the global gauge while the unitary gauge considered here is a local one. In SM, the gauge fixing for the leptonic multiplet is given by  $N = (0, 1)^T$  with  $SU(2)_W \otimes U(1)_Y$  as followed from the electro-weak interactions,

$$\left. \begin{array}{l} \psi_{fL} = \begin{pmatrix} \nu_f \\ e_f \end{pmatrix}_L \\ \psi_{fR} = e_{fR} \end{array} \right\}, \quad (20)$$

where the parity is defined by the projection operator  $(1 \pm \gamma^5)$ . The + and - signatures denote the left ( $L$ ) and right ( $R$ )-handedness of the particles respectively. In equation (20),  $f$  represents the family of leptons i.e.  $f = (e, \mu, \tau)$ . For instance, the masses of the first generation leptons (i.e. electron and its corresponding neutrino) are given as follows,

$$\left. \begin{array}{l} M_e = G v \\ M_\nu = 0 \end{array} \right\}, \quad (21)$$

while the masses of the gauge-bosons are defined as below,

$$\left. \begin{array}{l} M_W = \sqrt{\pi} g_2 v = \left[ \frac{\pi \alpha}{\sqrt{2} G_F \sin^2 \vartheta_w} \right]^{\frac{1}{2}} = \frac{37.3 GeV}{\sin \vartheta_w} \\ M_Z = \frac{M_W}{c_w} > M_W \end{array} \right\}, \quad (22)$$

where  $G_F \simeq 1.166 \times 10^{-5} \text{GeV}^{-2}$  and  $\alpha \simeq 1/137$  are the Fermi and Sommerfeld structure constants, respectively. However, the Weinberg term  $c_w = \cos \vartheta_w$  (for the mixing of  $Z^0$  with the  $W^\pm$  and  $A$ ) may be defined in terms of the coupling constant of the hypercharge in the following form,

$$g_2 \sin \vartheta_w = g_1 \cos \vartheta_w = e. \quad (23)$$

The measurements for the Weinberg mixing angle ( $\vartheta_w$ ) within SM lead to the following approximate values,

$$\left. \begin{aligned} \sin^2 \vartheta_w &\simeq 0.23 \\ \cos^2 \vartheta_w &\simeq 0.77 \end{aligned} \right\}. \quad (24)$$

The Weinberg mixing angle not only relates the masses of  $W^\pm$  and  $Z^0$  bosons, but also gives a relationship among the electromagnetic ( $e$ ), charged weak ( $g_2$ ) and neutral ( $g_1$ ) couplings and ultimately leads to the following approximate values of mass for  $W$  and  $Z$  bosons,

$$\left. \begin{aligned} M_W &\simeq 78 \text{ GeV} \\ M_Z &\simeq 89 \text{ GeV} \end{aligned} \right\}. \quad (25)$$

However, the baryonic matter field is given in terms of the doublets where  $f$  denotes the different generations of quarks (while in QCD, it counts the flavor with the color-triplet of  $SU(3)_C$ ). This interaction is given by a new enlargement term in the Lagrangian (which is necessary to generate the mass of fermions via Higgs Mechanism) as below,

$$\begin{aligned} \mathcal{L}(\phi, \psi) &= -G_f (\bar{\psi}^A \phi^{\dagger a} \hat{x} \psi_{aA} + \bar{\psi}^{aA} \hat{x}^\dagger \phi_a \psi_A) \\ &\equiv -M_f (\bar{\psi}^A N^{\dagger a} \psi_{aA} + \bar{\psi}^{aA} N_a \psi_A). \end{aligned} \quad (26)$$

The propagator for the exchanged boson (i.e. Higgs boson) via the Higgs interaction of two fermions turns out to be in the lowest order of the amplitude the same as derived from a Yukawa potential (i.e. a screened Coulomb potential). The propagator or Green function of such Klein-Gordon equation of a massive particle itself is enough to demonstrate that the Higgs interaction is of Yukawa-type. In fact, the scalar field ( $\phi_a$ ) couples with fermions ( $\psi_A$ ) through the Yukawa matrix  $\hat{x}$ , and the mass of the fermions may be then given as  $M_f = G_f v$ , and it is worth to notice that the equation (21) is a special case of it. Such Higgs-coupling to the fermions is model-dependent, although their form is often constrained by discrete symmetries imposed in order to avoid 3-level flavor changing neutral currents mediated by the Higgs exchange. However, to have a more accurate picture, the quantum mechanical radiative corrections are to be added in order to have an effective potential  $V_{eff}(\phi)$ . Since the coupling is also dependent on the effective mass of the field, the  $\lambda \mu^2 \phi^2$  and  $\lambda^2 \phi^4$  terms from a vacuum energy contribution are caused by vacuum fluctuations of the  $\phi$ -field and must be incorporated in the system to have a correct physical description. Furthermore, there are additional quantum gravitational contributions and temperature dependence so

that  $V_{eff}(\phi) \rightarrow V_{eff}(\phi, T) \sim V_{eff}(\phi) + M^2(\phi) T^2 - T^4$ . As a consequence, symmetry must be restored at high energies (or temperatures), especially in the primordial universe [21], which is contrary to the present state of the universe. The symmetry breakdown through the cooling of the universe after the Big Bang, in turn, provokes the appearance of the four well-known elementary interactions. In this context, though, it is an open question if there are more than one Higgs particle; it would be necessary that at least two of them exist in usual unifying theories to occur SSB so that gauge bosons become massive at different energy-scales as in GUT. Moreover, there is the symmetry breakdown of parity, too (which may be understood in terms of *axions*) [22, 23], the mechanism of which was claimed experimentally demonstrated shortly sometimes earlier during the year 2006 [24].

#### IV. SOME PHENOMENOLOGICAL ASPECTS

Though the SM explains and foresaw many aspects of nature proven by the well tested experiments, there is still a problem of special relevance which is popularly known as the *hierarchy* problem. The EW breaking scale related to the Higgs mass is expected too high in SM, and this seems quite unnatural to many physicists. This problem has apparently no solution within SM. If it can be solved, it can then signify non-elementarity of the Higgs fields. Indeed, if they are elementary, then there must be a symmetry protecting such fields from a large radiative correction to their masses [25]. In order to do so, the first choice is to take the Higgs field as a composite structure containing only an effective field (in the way one can explain superconductivity as following Higgs Mechanism), and it then seems indeed possible to construct a renormalizable SM without the fundamental Higgs scalar field [26]. However, the second choice is to supersymmetrize the SM, which leads to the possibility to cancel the unphysical quadratic divergences in the theory; in this way, it is possible to provide an answer to the *hierarchy* problem between the EW and Planck mass scale. The supersymmetric version of SM may, therefore, be an important tool to stabilize the *hierarchy* against the quantum corrections. As such, in the MSSM with the radiative EW symmetry breaking, the stability of the Higgs potential leads to the mass generation around the EW scale. In such SuSy versions one uses two doublets for the Higgs field as follows,

$$\Phi_1 = \begin{pmatrix} \phi_1^{0*} \\ \phi_1^- \end{pmatrix}; \quad \Phi_2 = \begin{pmatrix} \phi_2^+ \\ \phi_0^0 \end{pmatrix}, \quad (27)$$

to generate the mass of up and down fermions. The most general potential [3] for this purpose is of the following form,

$$\begin{aligned}
V(\Phi_1 \Phi_2) = & M_{11}^2 \Phi_1^\dagger \Phi_1 + M_{22}^2 \Phi_2^\dagger \Phi_2 - [M_{12}^2 \Phi_1^\dagger \Phi_2] + \frac{1}{2} \lambda_1 (\Phi_1^\dagger \Phi_1)^2 + \frac{1}{2} \lambda_2 (\Phi_2^\dagger \Phi_2)^2 \\
& + \lambda_3 (\Phi_1^\dagger \Phi_1) (\Phi_2^\dagger \Phi_2) + \lambda_4 (\Phi_1^\dagger \Phi_2) (\Phi_2^\dagger \Phi_1) + \frac{1}{2} \lambda_5 (\Phi_1^\dagger \Phi_2)^2 + [\lambda_6 (\Phi_1^\dagger \Phi_1) + \lambda_7 (\Phi_2^\dagger \Phi_2)] + h.c., \quad (28)
\end{aligned}$$

where the  $\lambda_6$  and  $\lambda_7$  are often dropped out in view of the cancelation by the following symmetry:

$$\Phi_1 \rightarrow -\Phi_1 \Rightarrow M_{12} = 0. \quad (29)$$

In view of the above-mentioned potential, the scalar field develops a non-degenerate VEV (with  $M_{ij}^2$  having at least one negative eigenvalue). The minimization of the potential leads to

$$\langle \Phi_1 \rangle = \begin{pmatrix} 0 \\ v_1 \end{pmatrix}; \quad \langle \Phi_2 \rangle = \begin{pmatrix} 0 \\ v_2 \end{pmatrix}, \quad (30)$$

which in turn defines,

$$v^2 = v_1^2 + v_2^2 = \frac{4M_W^2}{g^2} = (246 \text{ GeV})^2; \quad \tan \beta = \frac{v_2}{v_1}. \quad (31)$$

In this scenario, there are eight degrees of freedom in total including three Goldstone bosons ( $G^\pm, G^0$ ) those are absorbed by  $W^\pm$  and  $Z^0$  bosons. The remaining physical Higgs particles are two CP-even scalar particles ( $h^0$  and  $H^0$  with  $M_{h^0} < M_{H^0}$ ) (one is a CP-odd scalar  $A^0$  and the other is a charged Higgs pair  $H^\pm$ ). In fact, two of the neutral fields come from the real part  $\Re\phi_1^0$  and  $\Re\phi_2^0$  and the third actually belongs to the imaginary part of a linear combination of  $\Phi_1$  and  $\Phi_2$  [20]. However, the mass parameters  $M_{11}$  and  $M_{22}$  can be eliminated by minimizing the scalar potential. The resulting squared masses for the CP-odd and charged Higgs states are then given as follows:

$$M_{A^0}^2 = \frac{2M_{12}^2}{\sin 2\beta} - \frac{1}{2} v^2 (2\lambda_5 + \lambda_6 \tan^{-1} \beta + \lambda_7 \tan \beta), \quad (32)$$

$$M_{H^\pm}^2 = M_{A^0}^2 + \frac{1}{2} v^2 (\lambda_5 - \lambda_4). \quad (33)$$

The SuSy, therefore, couples the fermions and bosons in such a way that the scalar masses have two sources (given by  $v$ ) for their quadratic divergences, one from scalar loop which comes with a positive sign and another from a fermion loop with negative sign. The radiative corrections to the scalar masses can be controlled by canceling the contributions of the particles and their SuSy partners (i.e. s-particles) those come in the spectrum because of the breakdown of SuSy. Since SuSy is not exact, such cancelation might not be complete as the Higgs mass receives a contribution from the correction which is limited by the extent of SuSy-breaking. However, in the structure of this model, the quantum loop corrections would

induce the symmetry-breaking in a natural way, and they may be helpful in solving some other conceptual problems of SM [3]. Since the appearance of the top-quark at an extremely high energy-scale (in comparison to the one of all other quark flavors) could not be explained within the well established notions of SM, it might be understood as consequence of an unknown substructure of the theory. Therefore, the SM and MSSM would be only an effective field theory with another gauge force which is strong at  $SU(2) \otimes U(1)$  breaking-scale. On the other hand, in technicolor (TC) theory, the Higgs particles are not believed to be fundamental and the introduction of technifermions (a type of pre-quarks or preons) represent the quarks as composite particles with a new symmetry which is spontaneously broken when technifermions develop a dynamical mass (independently of any external or fundamental scalar fields. The developments of the TC theory, however, has not yet been fully able to suppress the possibility of such scalar fields completely out, so that an extended version of it would be required (i.e. ETC) [3]. Furthermore, another interesting fact is the heavy mass of the top-quarks, which leads to some models claiming that such a mass may only be generated dynamically with so-called top condensation [25] where the Higgs particles might have a phenomenological presence with the  $t\bar{t}$  bound states of top-quarks [3, 12]). Next, and listing more, if one thinks about the existence of Higgs particles in terms of the developments in LQG, the existence of the Higgs particles can be questioned because in LQG the particles are derived though a preon-inspired (Helon) model as excitations of the discrete space-time. Further, the Helon model does not offer a *preon* for Higgs (as a *braid* of space-time) so that it lacks of the same until the model indeed finds an expansion with a Higgs *braid* [27]. On the other hand, the phenomenological nature of Higgs (which might also explain other problems as the impossibility to measure the gravitational constant  $G$  exactly [28]) might also be a consequence of a more profound coupling as with gravitation. In view of the fact that the Higgs particles couple gravitationally within the SM [10, 11, 29], the consequences are discussed in detail in the next section.

## V. THE GRAVITATIONAL-LIKE INTERACTIONS AND HIGGS MECHANISM

The general relativistic models with a scalar field coupled to the tensor field of GR are conformally equivalent to the multi-dimensional models, and using Jordan's isomorphy theorem, the projective spaces (like in the Kaluza-Klein's theory) may be reduced to the usual Riemannian 4-*dim* spaces [30]. Such scalar field in the metric first appeared in Jordan's theory [31] and manifests itself as *dilaton*, *radion* or *gravi-scalar* for different cases, and those in fact correspond to a scalar field added to GR in a particular model [32]. The gravitational constant  $G$  is then replaced by the reciprocal of the average value of a scalar field through which the strength of gravity can be varied (thus breaking the SEP), as was first introduced by Brans and Dicke [33] by coupling a scalar field with the curvature scalar in the Lagrangian  $\mathcal{L}$ .

However, a more general covariant theory of gravitation can accommodate a massive scalar field in addition to the massless tensor field [34, 35] so that a generalized version of the Jordan-Brans-Dicke (JBD) theory with massive scalar fields can be derived [36]. It is worth mentioning that Zee was the first who incorporated the concept of SSB in the STT of gravitation [37]. It represents a special case of the so-called *Bergmann-Wagoner* class of STTs [38, 39]. The latter is more general than that of the JBD class alone (where  $\omega = \text{const}$  and  $\Lambda(\phi) = 0$ ), because of the dependence of the coupling term  $\omega$  on the scalar field and of an appearing cosmological function. In Zee's approach, the function  $\Lambda(\phi)$  depends on a symmetry breaking potential  $V(\phi)$ , and it is therefore quite reasonable to consider a coupling of Higgs particles with those which acquire mass through a short-ranged gravitational-like interaction within SM [10, 11, 29]. Such a model is compatible with Einstein's ideas to the Mach principle [40]. The simplest Higgs field model beyond the SM consists of a single particle which only interacts with the Higgs sector of SM. With a fundamental gauge-invariant construction block  $\phi^\dagger\phi$ , the simplest coupling of a particle to the Higgs field may be defined as  $\lambda\phi^\dagger\phi X$  where  $X$  is a scalar field. The Higgs field develops a VEV and, after shifting it, the vertex leads to a mixing between the scalar and the Higgs field, which may give rise to new effects those do not involve the scalar explicitly [9]. The X-field may not be considered as fundamental, but an effective description of an underlying dynamical mechanism is possible through its connection to the technicolor theories [9] (i.e. alternatively to a connection between the gravity and Higgs sector). In fact, both the gravity and Higgs particles possess some universal characteristics, and such a commonality leads to a relation between the Higgs sector and gravity which is popularly termed as the Higgs field gravity [11]. Further, there may be a similarity between  $X$  and the hypothetical graviton since both are the singlets under the gauge group [10]. They have no coupling to the ordinary matter and therefore have experimental constraints for their observations. One can

even argue about their absence from the theory because they can have a bare mass term which can be made to be of the order of the Planck mass and that makes these fields invisible. However, one can assume that all the masses including that of the Planck mass are given by SSB processes in nature. In this case, there is a *hierarchy* of mass scales  $M_P \gg v$ . With these similarities,  $X$  can be considered to be essentially the graviton and may be identified with the curvature scalar  $\mathcal{R}$  [10]. Moreover, this possibility may be used to explain the naturalness problem, especially since other candidates as top quark condensation or technicolor have not functioned well so far and supersymmetry doubles the spectrum of elementary particles replacing Bose (Fermi) degrees of freedom with Fermi (Bose) degrees of freedom and with all supersymmetric particles which are by now beyond physical reality. However, the cut-off of the theory at which the Higgs mass is expected may not be so large and of the order of the weak scale [12]. The Higgs particles therefore seem to couple naturally to the gravitation, and a STT of gravitation with a general form of Higgs field for symmetry breaking can indeed be derived within SM [41, 42]. Moreover, Higgs may be explained as a phenomenological appearance of the polarization of the vacuum since it leads to a cosmological term which may be identified in terms of the Higgs potential with a functional coupling parameter  $G$  [41, 42]. In such STTs the scalar field  $\phi$  may behave similarly to a cosmon [12]. The Higgs field may, therefore, also contribute in cosmological range as a part of the Cold Dark Matter (CDM) because of the functional nature of the coupling  $G$  and the self-interacting DM (SIDM) [43]-[50]. However, the unification of gravitation with the SM and GUT using Higgs field may also explain Inflation for baryogenesis and solve the flatness problem. The scalar fields and Higgs Mechanism lead to various inflationary models in cosmology where the cosmological constant produces the inflationary expansion of the universe [51]-[53]. Within the original, *old* inflation [54], the scalar field should tunnel from its false vacuum to the minimal value  $v$  while, on the other hand, in the *new* inflation it rolls slowly from  $\phi \ll v$  to  $v$  and then oscillates near to it. However, in case of the *chaotic* inflation, the rolling-over is explained in the range from  $\phi \gg v$  to  $v$  [21]. The new inflation can lead in a symmetry-broken STT to a deflation epoch before the expansion, and then a fine-tuning is needed for the universe not to collapse in a singularity again. However, for the chaotic inflation, which seems to be the most natural form of inflation not only within theories of induced gravity with Higgs fields but in general models [55], too, a singularity at the beginning of time is not needed because of a breaking of the Hawking-Penrose energy condition [56]-[58]. In fact, this is possible for sufficiently large negative pressures which are possible as a consequence of Yukawa-type interactions [59] that might play an important role in early stages of the universe [60]. Therefore, the chaotic inflation is precluded in general not by a singularity of a Big Bang but by a so-called Big Bounce, having its signatures within

LQG [61]. In particular, in a theory of induced gravity with Higgs mechanism [49], after inflation, the Higgs potential might decay in to the baryons and leptons with the oscillations from the Higgs potential which might be interpreted in terms of the SIDM in form of the Higgs particles. The actual interactions of Higgs particles are only given by their coupling to the particles within SM and the field equation for the Higgs field [41] can be given as follows,

$$\phi_{;\mu}{}^{;\mu} + \mu^2 \phi + \frac{\lambda}{6} (\phi^\dagger \phi) \phi = -2G (\bar{\psi}_R \hat{x} \psi_L), \quad (34)$$

where  $\hat{x}$  is the Yukawa coupling operator which represents the coupling of the Higgs field to the fermions, and the subscripts  $L$  and  $R$  refer the left- and right-handed fermionic states of  $\psi$  respectively. In SM the source of the Higgs field are the particles that acquire mass through it, and the Lagrangian for the case of a coupling of the Higgs field to space-time curvature through the Ricci scalar ( $\mathcal{R}$ ) [41] is given in the following form,

$$\mathcal{L} = \left[ \frac{\check{\alpha}}{16\pi} \phi^\dagger \phi \mathcal{R} + \frac{1}{2} \phi_{;\mu}^\dagger \phi^{;\mu} - V(\phi) \right] + \mathcal{L}_M, \quad (35)$$

where the dimensionless gravitational coupling parameter  $\check{\alpha}$  can be interpreted as a remnant of a very strong interaction which is given by the ratio of Planck's mass to boson mass as  $\check{\alpha} \simeq (M_P/M_B)^2 \gg 1$  [10]. On the other hand,  $\check{\alpha}$  is coupled to the gravitational strength  $G$ , on which the redefined scalar field mass of the model (i.e.  $M_H$ ) is dependent. This mass is expected around a  $10^{-17}$  part of the value needed in SM and  $10^{-4}$  of the one in GUT under SU(5). It is even possible for the Higgs particles to decouple from the rest of the universe and interact only gravitationally. That is the case if the same  $\phi$  has a coupling to  $\mathcal{R}$  in  $\mathcal{L}_M$  for mass generation of the gauge bosons [42]. Within this scalar-tensor theory then, the Higgs field equations with coupling to  $\mathcal{R}$  and  $\phi$  ( $\sim$  SM) and only to  $\mathcal{R}$  ( $\sim$  GUT), respectively, are given below:

$$\phi_{;\mu}{}^{;\mu} - \frac{\check{\alpha}}{8\pi} \mathcal{R} \phi + \mu^2 \phi + \frac{\lambda}{6} (\phi^\dagger \phi) \phi = -2G (\bar{\psi}_R \hat{x} \psi_L), \quad (36)$$

$$\phi_{;\mu}{}^{;\mu} - \frac{\check{\alpha}}{8\pi} \mathcal{R} \phi + \mu^2 \phi + \frac{\lambda}{6} (\phi^\dagger \phi) \phi = 0. \quad (37)$$

The field equation (36) after the symmetry breakdown (with the excited Higgs field which satisfies  $(1 + \varphi)^2 = 1 + \xi$ ) acquires the following form,

$$\xi_{;\mu}{}^{;\mu} + M^{*2} \xi = \frac{1}{1 + 4\pi/3\check{\alpha}} \frac{8\pi\tilde{G}}{3} [T - \sqrt{1 + \xi} \bar{\psi} \hat{m} \psi], \quad (38)$$

where  $\tilde{G} = (\check{\alpha} v^2)^{-1}$  is related to Newton's constant. The Higgs field mass is given by

$$M^{*2} = l^{-2} = \left[ \frac{16\pi\tilde{G}(\mu^4/\lambda)}{1 + \frac{4\pi}{3\check{\alpha}}} \right] = \left[ \frac{\frac{4\pi}{9\check{\alpha}} \lambda v^2}{1 + \frac{4\pi}{9\check{\alpha}}} \right], \quad (39)$$

which indicates that the Higgs field possesses a finite range defined by the length scale  $l$ .  $T$  is the trace of the energy-stress-tensor  $T_{\mu\nu}$  given in the following form,

$$T^{\mu\nu} = \frac{i}{2} \bar{\psi} \gamma_{L,R}^{(\mu} \psi^{;\nu)} + h.c. - \frac{1}{4\pi} (F^\mu{}_\lambda F^{\nu\lambda} - F_{\alpha\beta} F_a^{\alpha\beta} g^{\mu\nu}). \quad (40)$$

The field-strength tensor in equation (40) is defined as  $F_{\mu\nu} = -ig^{-1} [\mathcal{D}_\mu, \mathcal{D}_\nu]$  where  $\mathcal{D}_\mu$  is the covariant derivative. However, the generalized Dirac matrices  $\gamma^\mu = h_a^\mu \gamma^a$  in equation (40) satisfy the following relation,

$$\gamma^\mu \gamma^\nu + \gamma^\nu \gamma^\mu = 2g^{\mu\nu} \mathbf{1}. \quad (41)$$

The trace of the energy-stress tensor as mentioned in equation (38) is then given as follows,

$$T = \frac{i}{2} \bar{\psi} \gamma_{L,R}^\mu \psi_{;\mu} + h.c. = \sqrt{1 + \xi} \bar{\psi} \hat{m} \psi. \quad (42)$$

However, in view of the coupling of  $\phi$  with the matter-Lagrangian, the energy-stress tensor and source of Higgs particles cancel the contribution due to each other such that the Higgs particles are no longer able to be generated and interact only through the gravitational channel.

## VI. SEARCH FOR HIGGS BOSON AND CONSTRAINTS

The search for the Higgs boson is the premier goal for the high energy physicists as the SM without the Higgs boson (or at least Higgs Mechanism) is not manifestly consistent with nature. It is often said that the Higgs boson is the only missing piece of SM as the top-quarks are now already subject of experimental reality. The Higgs bosons could not be generated so far in the particle accelerators, although their practical reality in explaining mass is not questioned at all and has been proved in many ways. However, the fundamental character of Higgs bosons still demands more explanation, and due to the absence of experimental evidence, they lie in the category of yet to be discovered objects. From the point of view of the search for Higgs particles, SuSy-models are leading candidates (although no supersymmetric particles have been discovered so far), while TC-models do not contain Higgs particles at all and some gravitational theories are often interpreted as with Higgs particles only interacting gravitationally. The MSSM, having the particle spectrum of SM along with the corresponding superpartners and two Higgs doublets in order to produce mass, is consistent with supersymmetry as well avoiding the gauge anomalies due to the fermionic superpartners of bosons, stabilizing the search for Higgs mass. Using group renormalization techniques, MSSM Higgs masses have been calculated, and two specific bounds on it can be made for the case when the top-squark mixing is almost negligible and for the case when it is maximal. The assumption  $M_T = 175\text{GeV}$  and  $M_{\bar{T}} = 1\text{ TeV}$  leads to  $M_{h^0} \gtrsim 112$

GeV when the mixing is negligible, while maximal mixing produces the large value of  $M_{h^0} \gtrsim 125$  GeV. On the other hand, within a MSSM with explicit CP violation, to constrain CP phases in the MSSM, the measurements of thallium electric dipole moments (DPM) are used [62]. The present experimental constraints suggest that the lightest Higgs mass (say  $H_1$ ) has to lie in the range  $7 \text{ GeV} \lesssim M_{H_1} \lesssim 7.5 \text{ GeV}$  ( $\tan\beta \simeq 3$ ), or  $\lesssim 10 \text{ GeV}$  ( $3 \lesssim \tan\beta \lesssim 5$ ), assuming mild cancelations in the thallium EDM. In a scenario with explicit CP violation in MSSM, the lightest Higgs boson can be very light ( $M_{H_1} \lesssim 10 \text{ GeV}$ ), with the other two neutral Higgs bosons significantly heavier ( $M_{H_2, H_3} \gtrsim 100 \text{ GeV}$ ). Here, CP is explicitly broken at the loop level, and the three neutral MSSM Higgs mass eigenstates have no longer CP parities due to a CP-violating mixing between the scalar and pseudo-scalar neutral Higgs bosons. The lightest Higgs boson is mostly CP odd, and its production possibility at the Large Electron-Positron Collider (LEP) is highly suppressed. The second-lightest Higgs ( $H_2$ ) at  $\sim 110$  GeV dominantly decays in  $H_1$ , which then decays in two  $b$  quarks and  $\tau$  leptons. This leads to a decay mode containing 6 jets in the final state [62] that was recovered with only low efficiency by the LEP2.

In the search for the Higgs boson, the mass of Higgs particles is a constraint, and searches for it started in the early 1980's with the LEP1, and without knowing all parameters, it was nearly impossible to know at which energy-scale to search. In this constraint, maximal mixing corresponds to an off-diagonal squark squared-mass that produces the largest value  $M_{h^0}$  with extremely large splitting in top-squark mass eigenstates. On the other hand, the weak scale SuSy predicts  $M_h \gtrsim 130$  GeV, all relatively in accordance with a possible Higgs mass of the order 114 GeV for which CERN presented possible positive results in September 2000 (this was achieved after delaying the shut-down of LEP and reducing the number of collisions for the Higgs search to get additional energy and work over the original capacities of the collider). However, the experiments were forced to stop for further improvement in the accelerator (towards the new Lepton-Hadron Collider (LHC)) and such results could not be achieved once again by other laboratory groups. Further, the updates presented in 2001 lessened the confidence and more thorough analysis reduced the statistical significance of the data to almost nothing. However, with  $M_T$  known, at least another parameter is given in the theory, i.e.  $\tan\beta = v_2/v_1 \approx 1$  for all SuSy energy-scales [3]. Here,  $v_i$ ,  $i = 1, 2$  are the ground state values of each Higgs doublet needed for SuSy. In fact, the search for Higgs uses different possible decaying processes, and especially in the LEP, the decaying processes of electron-positron collisions produce  $WW$ ,  $ZZ$  and  $\gamma\gamma$ -pairs in most of the cases, given in the following form [63],

$$\left. \begin{array}{l} e^+e^- \rightarrow W^+W^- \\ e^+e^- \rightarrow ZZ \\ e^+e^- \rightarrow W^+W^-\gamma \\ e^+e^- \rightarrow \gamma\gamma \end{array} \right\}. \quad (43)$$

There are also other possible channels where hadrons or heavier lepton-pairs are seen in  $e^+e^-$  collisions as given below,

$$\left. \begin{array}{l} e^+e^- \rightarrow e^+e^-q\bar{q} \\ e^+e^- \rightarrow q\bar{q}(\gamma) \\ e^+e^- \rightarrow \mu^+\mu^- \end{array} \right\}. \quad (44)$$

Nevertheless, in experiments searching for Higgs particles, it is important to separate them from the  $HZ$ -channel (i.e.  $e^+e^- \rightarrow HZ$ ), and for this one has to pick out the  $H$  and  $Z$  decay products against the background of all other decay channels, although the cross-section is very small for the  $HZ$  channel with respect to hadron ones, and smaller for the greater masses of the Higgs particles. Especially the  $ZZ$  production is an irreducible background to  $ZH$  production. The  $Z$  bosons can decay in  $W$  bosons or, from an excited state, in other  $Z$  and Higgs bosons  $H$ . Then again,  $H$  is expected to decay into four jets with 60% possibility in the form of heavy hadrons,

$$\left. \begin{array}{l} H \rightarrow b\bar{b} \\ Z \rightarrow q\bar{q} \end{array} \right\}. \quad (45)$$

There is missing energy with 18% possibility, while a leptonic channel can exist having 6% possibility,

$$\left. \begin{array}{l} H \rightarrow b\bar{b} \\ Z \rightarrow l^+l^- \end{array} \right\}. \quad (46)$$

Moreover, another channel, which has 9% possibility, is the  $\tau$ -channel,

$$\left. \begin{array}{l} H \rightarrow b\bar{b}(\tau^+\tau^-) \\ Z \rightarrow \tau^+\tau^-(q\bar{q}) \end{array} \right\}. \quad (47)$$

Thus, experiments searching for  $Z$  boson events are therefore accompanied by a pair of bottom-quarks which all together have enough energy to come from a very heavy object (the Higgs boson candidates). Then, the total number of events in all decaying channels are to be compared against the total number expected in the theory along with the measured particle energy against the machine performance at all time-events to be sure that the changes in the accelerator energy and collision rate do not affect the interpretation. The cross-section for  $HZ$  channels is meager and dependent on the mass of the Higgs particles. For a Higgs mass of  $M_H = 110$  GeV, the cross-section for  $e^+e^- \rightarrow HZ$ -decays is already smaller than for  $ZZ$  ones, and decreasing for higher masses. For energies greater than 110 GeV, the cross-section of  $e^+e^- \rightarrow e^+e^-q\bar{q}$ -decays is the biggest, in the scale of  $10^4 \text{ pb}$  and increasing, while  $e^+e^- \rightarrow q\bar{q}(\gamma)$  is the next one, around  $10^3 - 10^2 \text{ pb}$  and increasing [63]. The cross-sections of a decay in muons  $\mu^+\mu^-(\gamma)$  and in  $\gamma$ -rays are almost equal. The decay channel in weakons  $e^+e^- \rightarrow W^+W^-$  ascends rapidly in energies higher than around 140 GeV, and it is almost constant after around 170 GeV with a cross-section something larger than for  $WW$  and  $\gamma\gamma$  in

a cross-section scale of  $10pb$ . The decay channel in  $ZZ$  is much weaker but perceivably higher than zero, around  $1pb$  after energies in the scale of  $180GeV$ . The same for the shorter  $W^+W^-\gamma$  decay channel. The  $HZ$  channel is expected even weaker than the last ones and perceivably unlike zero only after around  $220GeV$ , in a cross-section scale of  $10^{-1}pb$ . For the decay channels to be analyzed, it is important to detect the  $Z$  decays. Further, OPAL also detects  $Z \rightarrow e^+e^-$  events. The electron pair events have low multiplicity and electrons are identified by a track in the central detector and a large energy deposit in the electromagnetic calorimeter,  $E/p = 1$ . The  $Z \rightarrow \mu^+\mu^-$  events are also analyzed in L3 where the muons penetrate the entire detector and let a small amount of energy in the calorimeters. The L3 emphasizes lepton and photon ID with a precise BGO crystal ECAL and a large muon spectrometer. All detectors reside inside a  $r=6m$  solenoid with a magnetic field  $B=0.5T$ . The  $Z \rightarrow \tau^+\tau^-$  events are also detected by the DELPHI collaboration. Tau lepton-decays are dominated by 1 and 3 charged tracks, with or without neutrals, missing neutrino(s) and back-to-back very narrow *jets*. For them, DELPHI has an extra particle ID detector known as RICH. However, to detect heavy hadrons in nature, it is important to remind that they decay weakly, sometimes in leptons with long lifetime and characteristic masses and event shapes. For instance,  $b$  and  $c$  hadrons decay in to the leptons around 20% with high momentum  $p$ . The electrons are then ionized in tracking chambers while the muons match between the central track and muon chambers. Moreover, the leptons give charge to the decaying hadron as in  $e^+e^- \rightarrow Z \rightarrow b\bar{b}$  in L3. However, in the LHC a Higgs mass of up to twice of the  $Z$  boson mass might be measured. The production mode is based on partonic processes, as in the Tevatron, and the greatest rate should come from gluon fusion to form a Higgs particle ( $gg \rightarrow H$ ) via an intermediate top-quark loop where the gluons produce a virtual top-quark pair which couples to the Higgs particles. Furthermore, the alternatives are the channels of hadronic jets, with a richer kinematic structure of the events, which should allow refined cuts increasing the signal-to-background ratio. The latter channels are the quark-gluon scattering ( $q(\bar{q})g \rightarrow q\bar{q}H$ ) and the quark-antiquark annihilation ( $q\bar{q} \rightarrow gH$ ), both dominated by loop-induced processes involving effective  $ggH$  and  $ggHZ$ -couplings [64]. Nevertheless, there is still the possibility of more decaying channels, and the generalizations of SM (for example in supersymmetric models) demands the existence of more possible decays with supersymmetric particles (such as through squark loops). Such a generalization might be needed with respect to the problems those are not solvable within SM and seem to be definitely secured within the supersymmetric version of SM (i.e. MSSM). However, experimental evidences for supersymmetric particles are important to sustain the physical reality of the theory as well as to clarify the reason of such heavy masses (if these particles really do exist). In fact, the self-consistency of SM to GUT at a scale of about  $10^{16}GeV$

requires a Higgs mass with the upper and lower bounds (2003) as given below [63],

$$130 GeV \lesssim M_H \lesssim 190 GeV, \quad (48)$$

and which was corrected in 2004 (after exact measurements of top quark mass) with the following value,

$$M_H \lesssim 250 GeV. \quad (49)$$

Such higher values of mass for the Higgs bosons make the theory non-perturbative while too low values make vacuum unstable [63]. From the experimental point of view and within the standard models, the Higgs masses  $M_H < 114GeV$  are excluded at least with 95 % of the what we call confidence level. The Fermi laboratory also announced an estimate of  $117GeV$  for the same in June 2004. However, the EW data strongly prefers the light Higgs bosons and according to the fit precision of all data, the most likely value of Higgs mass should be slightly below the limit set by the direct searches at LEP2, while the upper limit for Higgs mass lies around  $220GeV$  at 95% of confidence level [65], which is given by equation (49) after making all the relevant corrections. Moreover, it would be quite interesting to look for an existing theoretical prediction of a Higgs mass around  $170GeV$  [66], which comes from an effective unified theory of SM based on noncommutative geometry and with neutrino-mixing coupled to gravity. The Higgs boson mass is, however, basically unrestricted, but even there are even indications from lattice calculations that the simplest version of EW interaction is inconsistent unless  $M_H \lesssim 700GeV$  [63]. With such a mass higher than  $800GeV$ , the Higgs bosons would be strongly interacting, so that many new signals would appear in the Higgs boson scenario. There are also some evidences for the indirect searches of Higgs scalar in Ultra High Energy (UHE) cosmic rays interactions [67]. *Where the Higgs particles do appear ?* is of course an open question, but an important puzzle to solve in the elementary particle physics, and it may be consisting of some new conjectures which have still to come in modern physics.

## VII. CONCLUDING REMARKS

The SM of modern elementary particle physics provides a concise and accurate description of all fundamental interactions except gravitation. The answer of the fundamental problem which allows the elementary particles to become heavy is now addressed in terms of the Higgs boson in SM, which is quite unlike either a matter or a force particle. The Higgs Mechanism is, therefore, a powerful tool of modern particle physics which makes the models mathematically consistent and able to explain the nature of fundamental interactions in a manifest way. The bosons and fermions are believed to gain mass through a phase transition via Higgs Mechanism. In this way, the particles are able to be

coupled with experiments, and a theoretical explanation may be given of how the mass generation takes place. Nevertheless, the Higgs particles, belonging to the Higgs field, are still not experimental reality and need to be observed to make any model complete. In the same way, the SM might be needed to be generalized consistently in view of the different hues of unification schemes and other models, viz GUT, SuSy, TC, STT (with or without Higgs Mechanism). One more possibility to answer the problem comes from the LQG, which seems to explain the general nature of all particles in space-time.

The Higgs particles interact in a gravitative and Yukawa form, but their nature is still not completely understood. Their fundamental existence is not a fact until and unless they are observed in high energy experiments such that the SSB could finally be believed to be the natural process of the mass generation mechanism. On the other hand, the Higgs particles may turn out to couple only gravitationally with a possibility to be generated in the accelerators in form of the SIDM. The search for Higgs particles is a very important task in physics and it is believed that their mass would be

achievable with the future generation of high energy experiments (especially in those which are scheduled to start at LHC in near future). In a lucid way of speaking, the Higgs bosons are believed to have a mass of less than 250GeV and over 130GeV, and the current experimental status is that they are heavier than 114GeV. The search for the Higgs boson is still a matter of speculation in the absence of clear experimental evidences, and the detection of Higgs particles as a real observable particle in future will be a momentous occasion (*eureka moment*) in the world of elementary particle physics to certify the basic ideas of SSB for the mass generation in the universe.

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