Dual Meissner Effect and Dielectric QCD Vacuum

Hemwati Nandan[1],* Nils M. Bezares-Roder[2], and H. C. Chandola[3]
[1] *Centre for Theoretical Studies, Indian Institute of Technology, Kharagpur - 721 302, INDIA*
[2] *Institut für Theoretische Physik, Universität Ulm, Ulm 890 69, GERMANY and*
[3] *Department of Physics, Kumaun University, Nainital - 26301, INDIA*

Introduction

There exists a striking parallelism between a conventional superconductor and QCD vacuum where the magnetic condensation of the topological objects (monopoles and dyons) provides an effective non-perturbative description of the confinement mechanism [1]. The confinement in condensed vacuum manifests itself in terms of the formation of thin tubes of colour electric flux. In such magnetic superconductors, the dual (Abelian) potentials along with the field operators for the topological objects are the natural variables to describe the large-scale structure of QCD vacuum [1].
In the present article, using the action for a dual (magnetic) superconductor, we compare the strength of the dual Meissner effect (DME) and the dielectric parameters for the case of monopole and dyon condensation.

The Dual Meissner Effect

We consider the following action motivated from the Zwanziger formulation of dual QCD [1]-[3],

$$\mathcal{S}_{_{AHM}} = \int d^4x \left\{ -\frac{1}{4}\,(\partial_\mu \tilde{C}_\nu - \partial_\nu \tilde{C}_\mu)^2 + \left|\, (\,\partial_\mu - i\,Q\,\tilde{C}_\mu\,)\,\Phi \right|^2 + \lambda\,(|\Phi|^2 - \Phi_0^2)^2 \right\}, \tag{1}$$

where the Higgs scalar field Φ is dyonic in nature and $Q = (e^2 + g^2)^{1/2}$ where e and g represent the electric and magnetic charges respectively. The equations of motion for the dual gauge ($\tilde{C}_\mu$) and dyonic (Φ) fields which govern the dynamics of the dyon condensation in QCD vacuum, can now be easily derived. Considering $\partial_\mu \Phi^* = 0 = \partial_\mu \Phi$, the field equation for $\tilde{C}_\mu$ takes the form [4],

$$\left[\,\Box + m_V^2\,\right]\,\tilde{C}_\mu - \partial_\mu\,(\partial^\nu\,\tilde{C}_\nu) = 0\,, \tag{2}$$

where $m_V = \sqrt{2}\,Q\,\Phi_0$ is the mass of dual gauge field and the divergence of equation (2) leads to $\partial^\mu\,\tilde{C}_\mu = 0$. The massless dual gauge quantum which propagates in the dyonically condensed QCD vacuum then satisfies,

$$\Box\,\tilde{C}^\mu = J_s^\mu, \tag{3}$$

where, J_s^μ is the screening current that resides in the vacuum. Now comparing (2) and (3) with Lorentz condition, we have,

$$J_s^\mu = -\,m_V^2\,\tilde{C}^\mu\,, \tag{4}$$

which is a typical screening current condition in dual QCD and reduces to London equation for static case. In the present dual formalism of QCD, among the field contents (colour magnetic (**B**) and electric (**E**) fields), **E** satisfies $\nabla \times \mathbf{E} = \mathbf{J}_s$. Using such considerations, one can obtain,

$$\nabla^2\,\mathbf{E} - \nabla\,(\nabla\cdot\mathbf{E}) - m_V^2\,\mathbf{E} = 0\,. \tag{5}$$

The screening current also satisfy an equation similar to (5). With $\mathbf{E} \equiv (0, 0, E_z(x)\,)$, the equation (5) reduces to,

$$\mathbf{k}\,\{\,\partial_x^2\,E_z(x) - m_V^2\,E_z(x)\,\} = 0\,, \tag{6}$$

which has the following general solution,

$$E_z(x) = D_1\,\exp\,(\,-\,m_V\,x\,) + D_2\,\exp\,(\,m_V\,x\,), \tag{7}$$

where D_1 and D_2 are integration constants. Since $E_z(0) = E_0$ at $x = 0$ and E_z can

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*Electronic address: hnandan@cts.iitkgp.ernet.in

not increase to infinity far from x which leads to $D_1 = E_0$ and $D_2 = 0$. The colour electric field thus penetrates the vacuum up to a finite depth m_V^{-1}. Hence the DME and confinement of colour electric sources with the formation of a flux tube between a quark and anti-quark. For the case of monopole

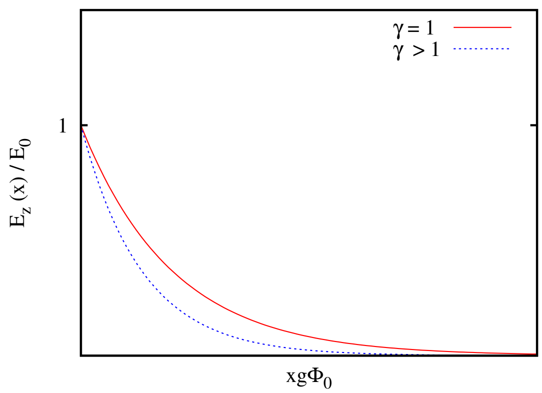

FIG. 1: Schematic view of DME for the case of monopole and dyon condensations ($\gamma = Q/g$).

condensation $m_V = m_g = \sqrt{2}\, g\, \Phi_0$ with $e = 0$, the colour electric field has somewhat higher penetration and therefore the electric field lines in a flux tube for this case are less squeezed in comparison to the case of dyon condensation.

The Dielectric Parameters

The vacuum polarisation and dielectric parameters are inherently connected through the polarisation tensor. In order to derive the polarisation tensor, one can translate the field Φ to a minimum energy position with its parameterisation as $\Phi = (\Phi_0 + \chi + i\eta)/\sqrt{2}$. The action $\mathcal{S}_{AHM}$ (1) can then be written in its following linearly approximated London form,

$$\int d^4x \left\{ -\frac{1}{4}\,(\partial_\mu \tilde{C}_\nu - \partial_\nu \tilde{C}_\mu)^2 + \frac{1}{2} m_V^2\, \tilde{C}_\mu \tilde{C}^\mu + \frac{1}{2}(\partial_\mu \chi)^2 + \frac{1}{2} m_\chi^2 \chi^2 + \frac{1}{2}(\partial_\mu \eta)^2 + \cdots \right\}, \tag{8}$$

where m_χ represents the mass of scalar χ. The mass spectrum here is similar to the AHM. As such with these considerations, the magnetic polarisation tensor [5] can be calculated in view of the usual Feynman rules as follows,

$$\tilde{\Pi}_{\mu\nu}(p) = (p_\mu p_\nu - p^2 g_{\mu\nu})\, \tilde{\Pi}\,(p^2, \Phi_0)\,, \tag{9}$$

where, the polarisation function is given as $\tilde{\Pi}\,(p^2, \Phi_0) = -m_V^2/p^2$ and the equation (9) remains valid for all values of momentum p. The polarisation tensor is in fact related to the dual gluon propagator as $\tilde{D}_{\mu\nu}(p) = \tilde{D}^0_{\mu\nu}\{1 + \tilde{\Pi}\,(p^2, \Phi_0)\}^{-1}$ where $\tilde{D}^0_{\mu\nu}$ is the bare dual gluon propagator. Using the polarisation function, the magnetic permeability may then be defined as below,

$$\mu\,(p^2, \Phi_0) = 1 - p^{-2}\; m_V^2. \tag{10}$$

In view of the relativistic invariance, the dielectric parameter can now be defined as $\epsilon\,(p^2, \Phi_0) \equiv \{\mu\,(p^2, \Phi_0)\}^{-1}$. The dielectric parameter of superconducting QCD vacuum vanishes with vanishing momenta so it behaves as a perfect dielectric medium. Since $m_V > m_g$, the dielectric parameter has always a greater value for the case of dyon condensation at a fixed momenta.

Conclusions

The dyon condensation is equally capable in describing the superconducting QCD vacuum as the monopole condensation and leads to DME and hence confinement, however with different strengths. The magnetic permeability in such vacuum rises to infinity with the vanishing momenta and therefore the dual QCD vacuum acts as a perfect dielectric medium in both the cases either with the dyon or monopole condensation.

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